

Exercises: Week 13

Computation in Algebra and Arithmetic

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1 Torsion points over \mathbf{Q}

Calculate $E(\mathbf{Q})_{\text{tors}}$ for each of the following elliptic curves.

- (a) $y^2 = x^3 - 2$.
- (b) $y^2 + xy + y = x^3 - x^2 - 14x + 29$.
- (c) $y^2 = x^3 + 8$.
- (d) $y^2 + xy = x^3 - 45x + 81$.
- (e) $y^2 = x^3 + 4$.
- (f) $y^2 + 43xy - 210y = x^3 - 210x^2$.

2 Weak Mordell-Weil

Use the results from subsection 10.4.3 to compute $E(\mathbf{Q})/2E(\mathbf{Q})$ for each of the following elliptic curves.

- (a) $E : y^2 = x(x - 1)(x + 3)$.
- (b) $E : y^2 = x(x - 12)(x - 36)$.