# Exercise Sheet 1.

Algebraic geometry

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Let $k$ be an algebraically closed field. Exercises below are from [Ellingsrud-Ottem].

**Q1** (1.8) Let $F_1, \cdots, F_r$ be homogenous polynomials in $k[x_1, \cdots, x_n]$ and let $X = Z(F_1, \cdots, F_r)$ be the closed algebraic subset they define. Show that $X$ is a cone with apex at the origin; that is, show that if $x$ is a point in $X$, the line joining $x$ to the origin lies entirely in $X$.

Draw a picture of $X$ when $r = 1, n = 2$ $F_1 = X_1X_2$.

**Q2** (1.10) Let $M_{n,m}$ be the space of $n \times m$-matrices with coefficients from $k$. It can be identified with the affine space $\mathbb{A}^{nm}$ with coordinates $x_{ij}$ where $1 \geq i \geq n$ and $1 \geq j \geq m$. Let $r$ be a natural number less than both $n$ and $m$, and let $W_r$ be the set of $n \times m$-matrices of rank at most $r$. Show that $W_r$ is a closed algebraic subset. Show that all the $W_r$'s are cones over the origin. Hint: Determinants are polynomials.

**Q3** (1.11) Let for every natural number $d \geq 2$ let $C_d \subset \mathbb{A}^d$ be the curve defined by the parametric representation $\phi(t) = (t, t^2, \cdots, t^n)$, and let $\mathfrak{a}$ be the ideal $\mathfrak{a} = (x_i - x_1x_{i-1} : 2 \geq i \geq d)$. Show that $C_d$ is a closed algebraic set, that $I(C_d) = \mathfrak{a}$ and that $\mathfrak{a}$ is a prime ideal.