

Exercise Sheet 10.

Algebraic geometry

04.05.2022

Let k be an algebraically closed field.

Q1 Lüroth theorem in algebra says that if L is a subfield of $k(t)$ such that L/k has transcendence degree 1, then there exists an element $u \in L$ such that $L = k(u)$. Reformulate it in the geometric language into a non-trivial statement about projective curves.

Q2 Let $\text{char}(k) \neq 2$. Show that the rational map

$$(x, y) : Z(y^2 - x^3 - x^2) \rightarrow \mathbb{P}^1$$

does not extend to a morphism (has no regular extension to 0)

Q3 Show that affine, projective, quasi-projective varieties over k are separated.