Exercise Sheet 11.

Algebraic geometry

11.05.2022

Let k be an algebraically closed field.

Q1 Consider the symmetric power map

$$\phi : \mathbb{P}^1 \times \cdots \times \mathbb{P}^1 \ (d \text{ times}) \ \to \mathbb{P}^d.$$

Prove that ϕ is a finite morphism.

Q2 Consider a map

$$\phi : \mathbb{P}^1 \to \mathbb{P}^1, z \mapsto [f(z) : g(z)]$$

for some $f, g \in k[z]$ with no common zeros. Compute the degree of ϕ (Hint: Gauss lemma).

Q3 (1) Suppose M is a B-module that is an increasing union of submodules M_i, with $M_0 = 0$, and every M_{i+1}/M_i is a free B-module. Show that M is a free B-module.

We say a B-algebra A satisfies ($\star$) if for each finitely generated A-module M, there exists a nonzero $f \in B$ such that M_f is a free B_f-module.

(2) Suppose B is a noetherian integral domain. Prove that B satisfies ($\star$).

(3) Suppose B is a noetherian integral domain and A is a finitely generated B-algebra satisfies ($\star$). Prove that $A[T]$ also satisfies ($\star$).