Exercise Sheet 12.

Algebraic geometry

18.05.2022

Let k be an algebraically closed field.

Q1 Describe $\mathrm{Spec}\mathbb{Z}[x]$ by considerting fibers of $\mathrm{Spec}\mathbb{Z}[x] \to \mathrm{Spec}\mathbb{Z}$.

Q2 Prove that $\mathrm{Spec}(R \times S)$ is homeomorphic to the disjoint union of $\mathrm{Spec}R$ and $\mathrm{Spec}S$.

Q3 Let X be an affine variety over an algebraically closed field k. Let $x \in X$ be a k-point and let m_x be the maximal ideal of the local ring $\mathcal{O}_{X,x}$ (here k-point means that $\mathcal{O}_{X,x}/\mathsf{m}_x \cong k$). A tangent vector at x is an element in $(\mathsf{m}_x/\mathsf{m}_x^2)^\vee$. Show that there is a bijection between k-algebra homomorphism $k[X] \to k[t]/t^2$ and pairs (x, v) where x is a k-point in X and v is a tangent vector at x.