Exercise Sheet 2.

Algebraic geometry

09.03.2022

Let k be an algebraically closed field.

Q1 Let X be an algebraic variety over k and Z be a closed subset of X. Consider the k-algebra sheaf defied as

$$\mathcal{O}_Z(U) = \{\text{continuous functions on } U \text{ that are locally restrictions of regular functions on } X\}.$$

Prove that $(Z, \mathcal{O}_Z)$ is an algebraic variety.

Q2 Show that the embedding $j : \mathbb{A}^2 - \{0\} \to \mathbb{A}^2$ induces an isomorphism of regular functions, i.e. sections of $\mathcal{O}_{\mathbb{A}^2}$.

Q3 Let X, Y be algebraic varieties and $U \subset X, V \subset Y$ open subsets. Assume there is an isomorphism of algebraic varieties $\phi : U \xrightarrow{\cong} V$ and define $W = X \sqcup Y / \sim$ where the equivalence relation identifies U and V via ϕ. Define $\mathcal{O}_W$ as the sheaf of functions whose pullbacks to both X and Y are regular. Show that $(W, \mathcal{O}_W)$ is an algebraic variety, with X and Y its open subvarieties.