Exercise Sheet 4.

Algebraic geometry

15.03.2022

Let k be an algebraically closed field.

Q1 Let X be an irreducible algebraic set and let $k(X) := \mathrm{Frac}(k[X])$ be its field of rational functions. Show that $k(X)$ can be defined as equivalence classes of regular functions defined on dense open subsets.

Q2 Show that any two different lines in $\mathbb{P}^2_k$ meet exactly in one point.

Q3 Assume $k = \overline{\mathbb{F}_p}$. Consider the Frobenius map $\mathrm{Frob} : \mathbb{A}^n_k \to \mathbb{A}^n_k$ that sends $(a_0, ..., a_n)$ to $(a_0^p, ..., a_n^p)$. Show that it is bijection but not an isomorphism. Explain why fixed points of Frob^m are the points with coordinates in $\mathbb{F}_{p^m}$.