

Exercise Sheet 6.

Algebraic geometry

30.03.2022

Let k be an algebraically closed field.

Q1 Let $k = \mathbb{C}$. Show that there exists a continuous map (in Zariski topology)

$$f : \mathbb{A}_k \rightarrow \mathbb{A}_k$$

which is not a regular function.

Q2 In the class we showed that for affine varieties X and Y , $\dim(X \times Y) = \dim X + \dim Y$. Deduce this formula for general algebraic varieties X, Y (possibly non-irreducible).

Q3 Let $f : X \rightarrow Y$ be a finite morphism. Show that f has finite fibers.

Q4 (from the [Rising sea, Example 11.3.9]) Let A be the ring defined by $k[x]_{(x)}[t]$. Show that there exists a codimension one prime ideal $\mathfrak{p} \subset A$ which has dimension zero. Note that $\dim A = 2$.