Exercise Sheet 6.

Algebraic geometry

06.04.2022

Let k be an algebraically closed field.

Q1[Ex 7.6.f] Let X be an algebraic variety, p be a point in X. Show that the differential map

$$d_p : \mathcal{O}_{X,p} \to \mathfrak{m}_p/\mathfrak{m}_p^2$$

that sends a function f to the equivalence class of $f - f(p)$, satisfies the Leibniz rule.

Q2[Ex 7.3(iii)] Compute singularities of the curve

$$V(X^m - Y^n) \subset \mathbb{A}_k^2\,,$$

over fields of arbitrary characteristics.

Q3[Ex 7.7] Show that the quadric cone

$$X = V(XY - Z^2) \subset \mathbb{A}_k^3$$

is a) singular and b) normal (i.e. its coordinate ring is integrally closed).

Q4[Ex 7.5] For

$$X = Z(f_1, \cdots, f_r) \subset \mathbb{A}_k^n\,,$$

we define the tangent bundle of X as the set

$$T(X) = \{(x,v) \in X \times \mathbb{A}_k^n : \sum_{i=1}^{n} \frac{\partial f_j}{\partial x_i}(x) \cdot v_i = 0, \forall j\}\,.$$

Show that $T(X)$ is an affine variety.