

Exercise Sheet 8.

Algebraic geometry

20.04.2022

Let k be an algebraically closed field.

Q1 Let $f, g : X \rightarrow Y$ be morphisms between algebraic varieties. Suppose Y is separated. Prove the following statements.

- a/ The graph $\Gamma_f = \{(x, f(x)) \in X \times Y\}$ is closed in $X \times Y$.
- b/ The set $\{x \in X : f(x) = g(x)\}$ is closed in X .
- c/ We further assume that X, Y are irreducible. Suppose there is a nonempty open subset $U \subset X$ such that $f|_U = g|_U$. Then $f = g$ on X .

Q2 An algebraic variety X is called rational if it is birationally equivalent to \mathbb{P}^n for some n .

- a/ Show that any conic in \mathbb{P}^2 is rational.
- b/ Show that cuspidal cubic $V(Y^2 - X^3) \subset \mathbb{A}^3$ is a rational.

Q3 Let X and Y be two algebraic varieties. Suppose there are points $P \in X$ and $Q \in Y$ such that the local rings $\mathcal{O}_{X,P}$ and $\mathcal{O}_{Y,Q}$ are isomorphic as k -algebras. Then show that there are open sets $U \subset X$ and $V \subset Y$ and an isomorphism of U to V which sends P to Q .

Q4 Prove that any two one dimensional irreducible algebraic varieties over k are homeomorphic. (Later we see that there are non-birational curves)