Introduction to AG 1

SS 2022

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§ Organization

- **Home page:**

https://metaphor.ethz.ch/x/2022/fs/401-3146-12L/

- **E-mail:** maria.yakerson@math.ethz.ch

- **Main source:**

„Introduction to algebraic varieties" G. Ellingsrud J. C. Ottem

https://www.uio.no/studier/emner/matnat/math/MAT4210/data/mastermat4210.pdf

intuition & pictures!

- **Additional sources:**

1. James Milne close to our course
   „Algebraic geometry"
2. Miles Reid elementary, many examples
   „Undergraduate algebraic geometry"
3. Ravi Vakil the best book for scheme theory
   „Foundations of algebraic geometry"

+ more on the homepage...

- How the lectures will go:

- the course is recorded, but don't let this stop you from asking & answering questions!
- notes will appear on the course page
- feel free to ask questions during/after lectures or in emails
- Comm Alg is a prerequisite (Kowalski's WS course, Atiyah, McDonald, Ellingsrud's notes)

- How the exercises will go:

- exercise sheet every week
- try to solve it at home
- solutions will be discussed in the class
- notes will appear on the course page

material from exercises goes in the exam!

- How the exam will go:

- oral exam 30 mins
- covers material from lectures & exercises
- questions mainly about:
  definitions, properties, constructions, theorems, proof ideas, examples
- try to always keep in mind examples of things you're talking about!

§ Introduction to algebraic geometry

Starting goal of AG: analyzing solutions of (systems of) polynomial equations.

We can't solve them in general, but we can study the geometry of their shapes, introduce their invariants, study their properties etc

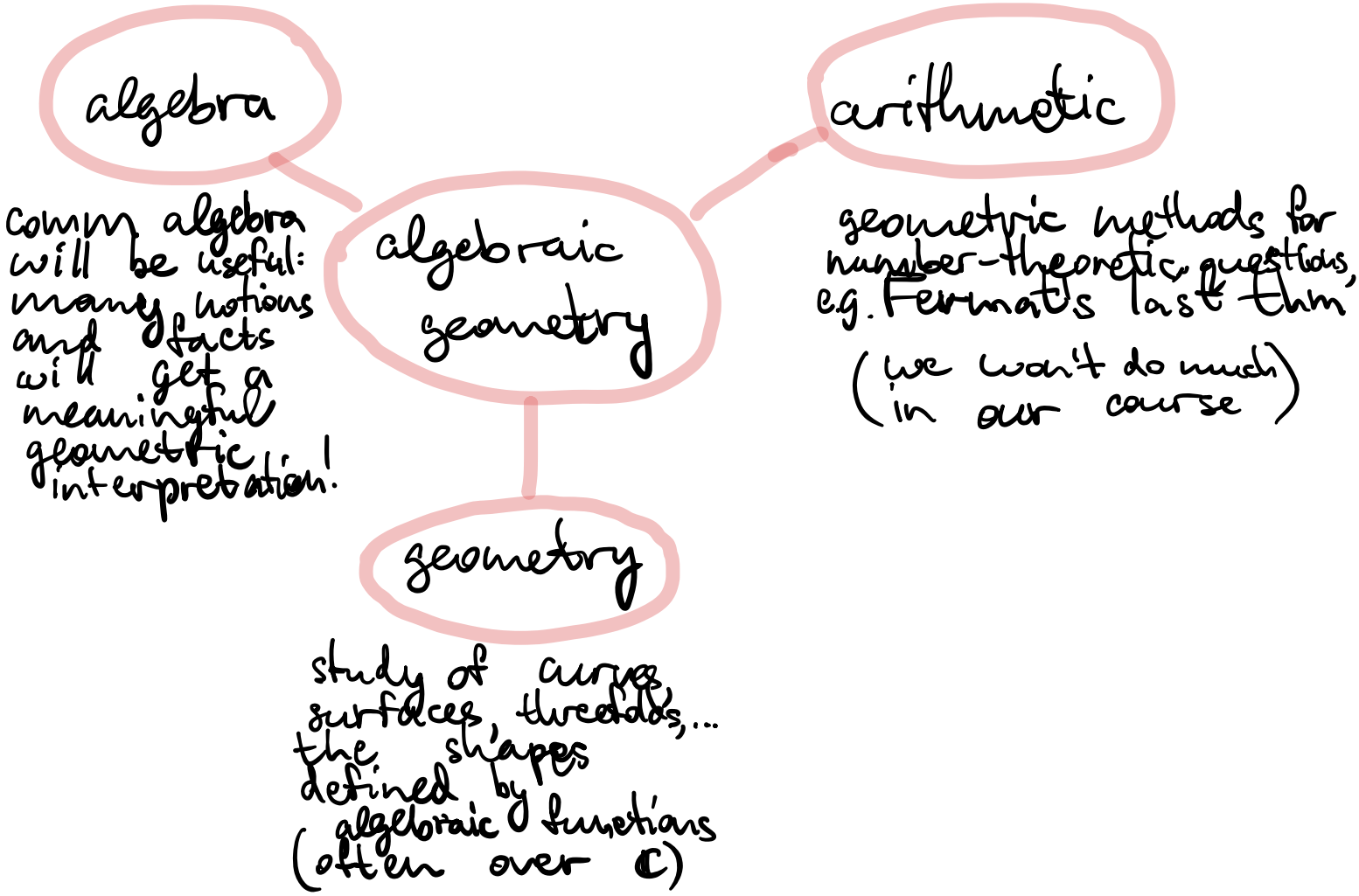
⇝ giant research area

In the course we will get a glimpse of it!

Most of the course: solutions over alg closed fields — varieties

In the ends of the course: arbitrary solutions and their generalizations — schemes

§ Big picture



AG also interacts a lot with:

- topology
- complex analysis
- differential geometry
- PDEs
- combinatorics
- ...

my research: apply methods of alg topology to objects in alg geometry

Millenium problems have a lot of alg geom!

1) Hodge conjecture: geometry vs topology of algebraic varieties (solutions of equations over $\mathbb{C}$)

2) Birch — Swinnerton-Dyer conjecture: arithmetic of elliptic curves (solutions of equations over $\mathbb{F}_p$ and $\mathbb{Q}$)

3) Riemann hypothesis: analytic statement that has an analogue in alg geom called Weil conjectures (solutions of equations over $\mathbb{F}_q$ and $\mathbb{C}$) they motivated the development of scheme theory (Grothendieck's school of alg geom) and were proved by Deligne.

We will hopefully see the statement of Weil conjectures at the end of the course!