Serie 1

Exercise 1. If Γ is a group, then $\ell^1(\Gamma)$ with the product defined via convolution and involution defined by $f^*(\gamma) := \overline{f(\gamma^{-1})}$ is an involutive Banach–algebra. Show that $\ell^1(\Gamma)$ is not a C^*–algebra unless $\Gamma = \{ e \}$.

Exercise 2. Let A be a Banach–algebra and define on $A_I := A \times \mathbb{C}$ the norm $\|(x, \lambda)\| = \|x\| + |\lambda|$. Show that A_I with this norm is a unital Banach–algebra.

Exercise 3. Suppose A is a unital $\mathbb{C}$–algebra with a Banach space norm $\|.\|$ for which the multiplication is continuous in each factor separately. Then there exists an equivalent norm $\|.\|'$ such that $\big(A, \|.\|'\big)$ is a unital Banach–algebra verifying $\|e\|' = 1$ and $\|xy\|' \leq \|x\|' \, \|y\|'$ for all $x, y \in A$.

Hint. For $x \in A$, consider the continuous operator $L_x : y \mapsto xy$ and its norm.