Serie 11

Exercise 1. Let $\mathbb{Z}/m\mathbb{Z}$ be the cyclic group of order m for $m \geq 1$. Show that the map

$$
\begin{array}{ccc}
\mathbb{Z}/m\mathbb{Z} & \longrightarrow & \widehat{\mathbb{Z}/m\mathbb{Z}}; \\
a & \longmapsto & \chi_a,
\end{array}
$$

where $\chi_a(x) = e^{\frac{2\pi i a x}{m}}$, is a group isomorphism.

Exercise 2. Show that $\hat{\mathbb{T}} \simeq \mathbb{Z}$ and $\hat{\mathbb{Z}} \simeq \mathbb{T}$ as locally compact abelian Hausdorff groups.

Exercise 3. Show that $\mathbb{R} \simeq \hat{\mathbb{R}}$ as locally compact abelian Hausdorff groups.