# Serie 13

**Exercise 1.** Show that for $G$ finite abelian $G \cong \hat{G}$.

**Exercise 2.** Show that any set $X_1, \dots, X_n$ of distinct characters are linearly independent over $\mathbb{C}$.

**Exercise 3.** Since $G$ finite implies $L^1(G) \cong \mathbb{C}[G]$, show directly that in a finite abelian group, for $f \in L^1(G)$ it holds

$$f(g) = \frac{1}{|G|} \sum_{\chi \in \hat{G}} \hat{f}(\chi)\chi(g).$$

Hence, the map $\mathbb{C}[G] \to \mathbb{C}[\hat{G}]$ that sends $f \mapsto \hat{f}$ is injective. Using the two first exercises, show that $f \mapsto \hat{f}$ is also surjective.