Serie 2

Exercise 1. If X is a locally compact and Hausdorff space, then for every $f \in C_0(X)$ the following equality holds

$$Sp_{C_0(X)}(f) = \overline{f(X)}.$$

Exercise 2 (Volterra Algebra)**.** Let $\mathcal{L}$ be the Lebesgues measure on $[0,1]$.

1. Show (using Fubini Theorem) that for any $f, g \in L^1([0,1])$,

$$f \star g(t) := \int_0^t f(t-x)g(x)d\mathcal{L}(x)$$

   exists for almost every $t \in [0,1]$ and that $f \star g \in L^1([0,1])$.

2. Show that $(f,g) \mapsto f \star g$ gives a Banach algebra structure on $L^1([0,1])$.
3. Show that for every $f \in L^1([0,1])$ it holds $Sp(f) = \{0\}$.

Hint. Let f_0 be the constant map at 1. Compute $f_0 \star \cdots \star f_0$ and observe that $\mathbb{C}[f_0]$, the subalgebra generated by f_0, is dense in $f \in L^1([0,1])$.