Serie 4

Exercise 1. Show that for any x in an involutive Banach algebra with involution \cdot^* it holds $\exp(x)^* = \exp(x^*)$.

Exercise 2. Let A be a unital Banach algebra with identity e and x an element in A. If λ is in the spectrum of x, then for any $a \in A$ the element $(x - \lambda e)a$ is not invertible.

Exercise 3. Find an example of Banach algebra such that its Guelfand transform is not surjective.

Hint. You can study the Banach algebras $C^1([0,1])$ or $\ell^1(\mathbb{Z})$ for examples.