Serie 5

Exercise 1. A matrix A with coefficient in $\mathbb{C}$ represents a normal operator if and only if there exists a unitary matrix u so that uAu^{-1} is diagonal.

Exercise 2. Show that the spectrum of δ_1 (characteristic map at 1) in $\ell^1(\mathbb{Z})$ is contained in $\mathbb{T} := \{ z \in \mathbb{C} \mid |z| = 1 \}$.

Hint. Use that $\widehat{\ell^1(\mathbb{Z})} \cong \mathrm{Hom}(\mathbb{Z}, \mathbb{T})$ and that for A a unital Banach algebra, $\hat{A}$ is compact and $Sp_A(x) = \hat{x}\big(\hat{A}\big)$.

Exercise 3. Find a unital Banach algebra with identity e and two element x, y so that x is not invertible but xy is invertible.

Hint. You can study the space of continuous operators $T : \ell^2(\mathbb{N}) \to \ell^2(\mathbb{N})$.