Serie 8

Exercise 1. Let X be a compact Hausdorff space and μ a positive regular measure on X. For every $f \in C(X)$ show that the multiplication operator defined on $L^2(X, \mu)$ by

$$M_f g(x) \colon f(x)g(x),\ f \in L^2(X, \mu),$$

has norm $\|M_f\| = \|f\|_\infty$ where $\|\cdot\|_\infty$ refers to the norm of f seen as an element of $L^\infty(X, \mu)$. Define also

$$A := \left\{ M_f \;\middle|\; f \in C(X) \right\} \subset \mathcal{L}\left(L^2(X, \mu)\right).$$

1. Show that A is a commutative sub–C^*–algebra of $\mathcal{L}\left(L^2(X, \mu)\right)$,
2. Taking into account that $\widehat{C(X)}$ can be identified with X, determine $\hat{A}$,
3. Determine the resolution of identity on $\hat{A}$ given by the Spectral Theorem.