

Machine Learning in Finance

Exercise sheet 6

Exercise 6.1 (Convex risk measure)

- (a) Recall the definition of convex risk measure on L^∞ . (Hint: monotone decreasing, convex, cash invariant)
- (b) Assume $l: \mathbb{R} \rightarrow \mathbb{R}$ is a loss function i.e. continuous, non-decreasing and convex. We define

$$\rho(X) = \inf_{w \in \mathbb{R}} \{w + \mathbb{E}[l(-X - w)]\}. \quad (1)$$

Prove that ρ define a convex risk measure.

- (c) Let $\lambda > 0$ and $l(x) = \exp(\lambda x) - \frac{1+\log(\lambda)}{\lambda}$. Solve (1) explicitly, write down the minimizer, and prove that

$$\rho(X) = \frac{1}{\lambda} \log \mathbb{E}[\exp(-\lambda X)] \quad (2)$$

which is the entropic risk measure.

- (d) Let $\alpha \in (0, 1)$, $l(x) = \frac{1}{1-\alpha} \max(x, 0)$. Solve (1) explicitly, write down the minimizer, and prove that

$$\rho(X) = \frac{1}{1-\alpha} \int_0^{1-\alpha} \text{VaR}_\gamma(X) d\gamma \quad (3)$$

which is the expected shortfall. VaR is the value at risk.

- (e) Is value at risk a convex risk measure on L^∞ ?

Exercise 6.2 (Indifference pricing) Let ρ be a convex risk measure and

$$\pi(X) = \inf_{\delta \in \mathcal{H}} \rho(X + (\delta \cdot S)_T - C_T(\delta)) \quad (4)$$

- (a) Recall the definition of indifference price p with π . Give financial interpretation of δ , \mathcal{H} , $(\delta \cdot S)$ and $C_T(\delta)$ above.
- (b) Suppose $C_T = 0$ and Z is attainable by $Z = p^* + (\delta^* \cdot S)$. Prove that $p(Z) = p^*$.
- (c) Suppose $C_T = 0$. Prove that the indifference price is bounded by super-hedging price.

Exercise 6.3 (Path dependence hedging) Code a deep path dependent hedging strategy, see exercise notebook.

References

- [1] Hans Buehler, Lukas Gonon, Josef Teichmann, and Ben Wood. Deep hedging. *Quantitative Finance*, 19(8):1271–1291, 2019.
- [2] Hans Föllmer and Alexander Schied. *Stochastic finance*. de Gruyter, 2016.
- [3] Blanka Horvath, Josef Teichmann, and Žan Žurič. Deep hedging under rough volatility. *Risks*, 9(7):138, 2021.