Modular Forms: Problem Sheet 6

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1. Show that if f is a non-zero weakly modular function of weight k, level Γ, and $g \in \mathrm{SL}_2(\mathbb{Z})$, we have

$$v_P(f|_k g) = v_{gP}(f)$$

   for all $P \in \mathrm{Cusps}(\Gamma)$.

2. Show that for the function $F(z) = \prod_{i=1}^{d}(f|_k g_i)(z)$ defined in the proof of Theorem 2.6.3, we have

$$V_{\Gamma'}(F) = \sum_{i=1}^{d} V_{\Gamma'}(f|_k g_i).$$

3. Let $F(z) = E_4(2z)$, so $F \in M_4(\Gamma_0(2))$.
   (a) Show that $F(\infty) = 1$ and $F(0) = \frac{1}{16}$.
   (b) Hence show that the subspace of $M_8(\Gamma_0(2))$ spanned by E_4^2, E_4F and F^2 is 3-dimensional, and contains a unique cusp form f with $a_1(f) = 1$. Calculate the q-expansion of this form as far as the q^3 term.
   (c) Use the valence formula and its corollaries to show that
      i. the functions $\{E_4^2, E_4F, F^2\}$ are a basis of $M_8(\Gamma_0(2))$,
      ii. $f(z) = \frac{\Delta(z)+256\Delta(2z)}{E_4(z)}$.