Only the exercises with an asterisk (*) will be corrected.

1.1. MC questions.

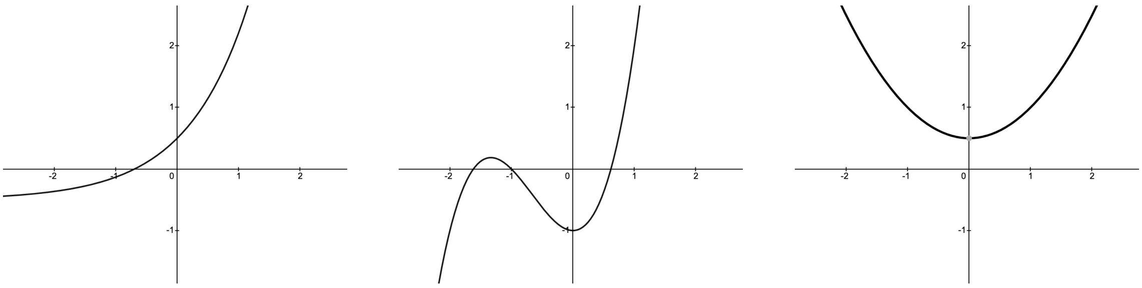
(a) How many solutions does the ODE

$$y'(x) = \frac{\sqrt{y(x)}}{1 - \operatorname{sgn}(y(x))}$$

have? Here, y is a function on $\mathbb{R}$ with values in $\mathbb{R}$.

- ☐ 0;
- ☐ 1;
- ☐ Infinitely many.

(b) Which graph corresponds to $(x, g(x))$, where $g\colon \mathbb{R} \to \mathbb{R}$ is a solution to the ODE $y'(x) = x$?



(a) (b) (c)

- ☐ (a)
- ☐ (b)
- ☐ (c)

*1.2. ODE classification.

For each of the following expressions, determine whether they are an ODE, and if yes, determine their order and whether they are linear/nonlinear, homogeneous/inhomogeneous.

(a) $(y'(x) + 1)^2 = y(x) + 1$.

(b) $a\, x^3\, y'''(x) +\ x\, y'(x) = 1$, where $a \in \mathbb{R}$.

(c) $y'(x) - y(3x) = y(2x)$.

(d) $y(x) = xy'(x) + f(y'(x))$, where $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable.

(e) $2xy'(x) + y(x) = e^x$.

1.3. Verifying solutions. Consider the ODE:

$$y'' + y' - 6y = 0,$$

where $y \colon \mathbb{R} \to \mathbb{R}$ is a twice continuously differentiable function.

(a) Verify that e^{-3x} and e^{2x} are solutions of the equation.

(b) Verify that $a\mathrm{e}^{-3x} + b\mathrm{e}^{2x}$ is again a solution of the equation for every $a, b \in \mathbb{R}$.

(c) Find all the (twice continuously differentiable) functions $\alpha : \mathbb{R} \to \mathbb{R}$ so that $\alpha(x)\mathrm{e}^{-3x} + \mathrm{e}^{2x}$ is a solution of the equation?

Hint: you may use the fact that the solutions to the equation $y'' - 5y' = 0$ are given by $y(x) = c_1 e^{5x} + c_2$ for $c_1, c_2 \in \mathbb{R}$.

***1.4. Finding ODE with the given solution.** Find an ODE of the specified order solved by the given function:

(a) $\varphi(t) = \frac{1}{t}$, of 1st order,

(b) $\varphi(t) = t \cos t$, of 2nd order,

(c) $\varphi(t) = \mathrm{e}^{t^3}$, of 1st order.