Only the exercises with an asterisk (*) will be corrected.

2.1. MC questions.

(a) Which of the following functions has the property that its arc length is equal to its area under the curve?

For a function $f\colon \mathbb{R} \to \mathbb{R}$, the arc length between 0 and x is given by

$$\int_0^x \sqrt{1+(f'(t))^2}\mathrm{d}t.$$

(a) $f(x) = \cosh x$, ☐
(b) $f(x) = x$ ☐
(c) $f(x) = e^x$ ☐
(d) $f(x) = \ln(x)$. ☐

(b) Choose the correct statement(s). Motivate your answers.

A particular solution of the ODE $y'' - 4y' - 12y = \sin(2x)$. is:

(a) $\frac{1}{40}\cos(2x) - \frac{1}{40}\sin 2x$ ☐
(b) $\frac{1}{15}\sin(2x)$ ☐
(c) $\frac{1}{20}\cos(x)$ ☐
(d) $\frac{1}{40}\cos(2x) - \frac{1}{20}\sin(2x)$. ☐

***2.2. Motion of a spring.** A piece of mass m connected to a coil spring that can stretch along its length. If $k > 0$ denotes the spring constant, the equation of motion of such system is given, according to Hooke and Newton's laws, by

$$m\ddot{x}(t) = -kx(t), \qquad (\dagger)$$

where $x = x(t)$ denotes the position in time of the piece of mass along the vertical direction. Call $\omega = \sqrt{\frac{k}{m}}$. Find the solution of (†):

(a) with initial position $x(0) = 0$ and initial velocity $\dot{x}(0) = -\omega$.

(b) with initial position $x(0) = 1$ and position at time $t = \frac{\pi}{2\omega}$: $x(\frac{\pi}{2\omega}) = 3$.

(c) Is is possible to find a solution so that $x(t) \to 0$ as $t \to +\infty$?

2.3. Projectile motion. Consider a ball being thrown from height $y_0 = 2\,\mathrm{m}$ at an angle $\theta = 30°$ with initial speed $v = 15\,\mathrm{m\,s^{-1}}$. At which horizontal distance from its starting point will the ball hit the ground?

You may assume that ball's trajectory $(x(t), y(t))$, where $x\colon \mathbb{R}_+ \to \mathbb{R}$ is its horizontal coordinate and $y\colon \mathbb{R}_+ \to \mathbb{R}$ is its vertical coordinate, satisfies the following ODE:

$$\ddot{x} = 0, \qquad \ddot{y} = -g. \tag{$\clubsuit$}$$

Here, $g = 9.81\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration.

***2.4. Radioactive decay.** Radioactive decay is the process by which an unstable atomic nucleus loses energy by radiation. It is described by the following equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N,$$

where $\lambda > 0$ is a positive constant and N is the amount of the radioactive material.

We define the half life T of a radioactive material as the time required for the half of the initial amount of radioactive material to decay.

Find the expression for the half life T in terms of the constant λ.