Only the exercises with an asterisk (*) will be corrected.

4.1. MC questions.

(a) Which of the following guesses give us a particular solution to the following ODE:

$$y'' - 2y' + 2y = \mathrm{e}^x \cos(x)?$$

- ☐ $y = c\mathrm{e}^x \cos(x)$ for a constant $c \in \mathbb{R}$
- ☐ $y = cx\mathrm{e}^x \cos(x)$ for a constant $c \in \mathbb{R}$
- ☐ $y = c_1 x\mathrm{e}^x \cos(x) + c_2 x\mathrm{e}^x \sin(x)$ for constants $c_1, c_2 \in \mathbb{R}$
- ☐ $y = c_1 \mathrm{e}^{(1+\mathrm{i})x} + c_2 \mathrm{e}^{(1-\mathrm{i})x}$ for constant $c_1, c_2 \in \mathbb{C}$
- ☐ $y = c_1 x\mathrm{e}^{(1+\mathrm{i})x} + c_2 x\mathrm{e}^{(1-\mathrm{i})x}$ for constant $c_1, c_2 \in \mathbb{C}$

(b) Which of the following statements are correct for $f\colon \mathbb{R}^n \to \mathbb{R}^m$, $g\colon \mathbb{R}^m \to \mathbb{R}$ and $x \in \mathbb{R}^n$?

- ☐ f is continuous at x if there exists a sequence $(x_k)_{k\in\mathbb{N}} \subseteq \mathbb{R}^n$, such that $f(x_k) \to f(x)$ for $k \to \infty$.
- ☐ The compositions $f \circ g$ and $g \circ f$ are always defined.
- ☐ If $g \circ f$ and f are continuous, then g is also continuous.

4.2. Inhomogeneous ODE.

Solve the following ODE:

$$y''' - 3y'' + 3y' - y = 4e^t.$$

***4.3. ODE change of variables.** Solve the following differential equation:

$$y' = (4x - y + 1)^2$$

Hint: use the substitution $u = 4x - y$.

***4.4. (In)homogeneous ODEs.** Determine the general (real) solutions of the following differential equations, for $x > 0$.

(a) $x^2y''(x) - 3xy'(x) + 5y(x) = 0.$

(b) $2xy'(x) - y(x) = \log(x).$

(c) $2y'' + 3y' + 10y = \sin(2x) + 1.$

Hint: For parts (a) and (b), consider the substitution $h(t) := y(e^t)$. If y is a solution to an equation above, then h solves a linear ODE with constant coefficients.