Only the exercises with an asterisk (*) will be corrected.

5.1. MC questions.

(a) Let $f : (-\frac{\pi}{2}, \frac{\pi}{2}) \times \mathbb{R} \to \mathbb{R}$ be given by $f(x,y) = \sin(y)^x$. Then for the partial derivatives of f it holds:

- ☐ $\frac{\partial f}{\partial x}(x,y) = x\sin(y)^{x-1}$ and $\frac{\partial f}{\partial y}(x,y) = \cos(y)^x$.
- ☐ $\frac{\partial f}{\partial x}(x,y) = \sin(y)^x \log(\sin(y))$ and $\frac{\partial f}{\partial y}(x,y) = x\cos(y)\sin(y)^{x-1}$.
- ☐ $\frac{\partial f}{\partial x}(x,y) = \log(1+|x|)\sin(y-x)$ and $\frac{\partial f}{\partial y}(x,y) = e^{\cos(y)}x$.

(b) On which set $M \subseteq \mathbb{R}^2$ does the function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) = |xy|$$

have both partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

- ☐ $M = \{(x,y) \in \mathbb{R}^2 \mid x \neq 0 \text{ und } y \neq 0\} \cup \{(0,0)\}$.
- ☐ $M = \mathbb{R}^2$.
- ☐ $M = \{(x,y) \in \mathbb{R}^2 \mid x \neq 0 \text{ und } y \neq 0\}$.

5.2. Limits in $\mathbb{R}^n$

(a) Let f be a function defined by $f(x,y) = \frac{y}{x-1}$ on the set $\{(x,y) \in \mathbb{R}^2 \mid x \neq 1\}$. Does the limit $\lim_{(x,y)\to(1,0)} f(x,y)$ exist? If it does, calculate it.

(b) Let f be a function defined by $f(x,y) = \frac{x^2y}{x^4+y^2}$ on the set $\mathbb{R}^2\backslash\{(0,0)\}$. Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If it does, calculate it.

(c) Let f be a function defined on the set $\{(x,y) \in \mathbb{R}^2 \mid x > 0 \text{ und } (x,y) \neq (1,0)\}$ by $f(x,y) = \frac{(x-1)^2\ln(x)}{(x-1)^2+y^2}$ Does the limit $\lim_{(x,y)\to(1,0)} f(x,y)$ exist? If it does, calculate it.

(d) Let f be a function defined by $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ on the set $\mathbb{R}^2\backslash\{(0,0)\}$. Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If it does, calculate it.

*5.3. Continuity in $\mathbb{R}^n$

(a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function given by

$$f(x,y) = \begin{cases} \frac{(x+y)y}{x^2+y^2} & \text{für } (x,y) \in \mathbb{R}^2 \backslash \{(0,0)\} \\ 0 & \text{für } (x,y) = (0,0). \end{cases}$$

Is f continuous?

(b) Let $f(x,y) = \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$ be defined on the set $\{(x,y) \in \mathbb{R}^2 \mid x,y > 0 \text{ und } x \neq y\}$. Can f be continuously extended to the set $\{(x,y) \in \mathbb{R}^2 \mid x,y \geq 0\}$?

(c) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & \text{für } (x,y) \in \mathbb{R}^2 \backslash \{(0,0)\} \\ 0 & \text{für } (x,y) = (0,0). \end{cases}$$

Is f continuous?

(d) Let $g : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$g(x,y) = \begin{cases} x\sin(1/y), & y \neq 0 \\ 0, & y = 0 \end{cases}$$

For which $(x,y) \in \mathbb{R}^2$ is g continuous?

***5.4. Partial derivatives.** Calculate all the partial derivatives of the following functions:

(a) $f(x,y) = x\,;$

(b) $f(x,y) = e^{xy}\,;$

(c) $f(x,y) = x^y\,;$

(d) $f(x,y) = \frac{x-y}{x^2+y^2}\,;$

(e) $f(x,y) = x^2 y \sin(xy)\,;$

(f) $f(x,y,z) = xy^2z^3\,.$