Only the exercises with an asterisk (*) will be corrected.

9.1. MC questions.

(a) Choose the correct statement. Motivate your answer.

Recall that a *critical point* of a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ is an $x_0 \in \mathbb{R}^n$ so that $df(x_0) = 0$. At such point, the tangent plane to the graph of f is:

- ☐ not defined
- ☐ horizontal (looking at $\mathbb{R}^3$ in the usual way with upward-pointing z-axis)
- ☐ vertical (looking at $\mathbb{R}^3$ in the usual way with upward-pointing z-axis)
- ☐ none of the above, in general.

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuously differentiable function and consider its restriction on the square $Q = [0, 1] \times [0, 1] \subset \mathbb{R}^2$.

- ☐ If f has a local maximum, minimum, or a saddle point at x_0 in Q, then $df(x_0) = 0$.
- ☐ Let $x_0 \in Q$ be a point such that $df(x_0) = 0$, then f has a local max/min/saddle at x_0.

*9.2. Critical points

Find the critical points of the following functions and determine whether they are local minima, local maxima, or saddle points.

(a) $f\colon \mathbb{R}^2 \cap \{(x, y) \mid x > 0,\ y > 0\} \to \mathbb{R}, \qquad f(x, y) = \dfrac{y}{2x} + \dfrac{x-1}{y^2}$

(b) $f\colon \mathbb{R}^2 \to \mathbb{R}, \qquad f(x, y) = x^3 + y^3 + 3xy$

(c) $f\colon \mathbb{R}^3 \to \mathbb{R}, \qquad f(x, y, z) = x^2 + y^2 + z^2 + 2xyz.$

9.3. Centre of mass.

Consider a system of N particles in $\mathbb{R}^n$, that is N points $a_1, \ldots, a_N$ with masses $m_1, \ldots, m_N$. Prove that the expression

$$I(x) = \sum_{i=1}^{N} m_i |x - a_i|^2$$

has a unique global minimum, and find it explicitly. Such point C is called the *centre of mass* of the system.

*9.4. Optimisation problem with constraints.

Determine the global extrema of the function

$$f(x, y) = x^2 + y^2 + 7x - 2y$$

on the set $D = \{(x, y) \mid x \geq 0,\ y \geq 0,\ 3x + y \leq 3\}$.

Hint: Try drawing D and consider the interior of D and the boundary separately.