0.1.

(a) Determine the solutions to the differential equation

$$y''' - 4y'' + 6y' = 0$$

satisfying the conditions

$$\lim_{t\to-\infty} y(t) = 0 \quad \text{and} \quad y'(0) = 0.$$

(b) Determine a solution of the equation

$$y''' - 4y'' + 6y' = e^{2t} + 9t^2.$$

Hint: since the equation is linear, you may divide the right-hand side in 2 parts and then sum the solutions (superposition principle).

(c) Determine the solution of the problem

$$\begin{cases} y = 2t^2 y' & \text{for } t \geq 1, \\ y(1) = 1. \end{cases}$$

0.2. Determine the critical points of the following function and whether they are local maximum, minimum, or saddle points:

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = y(x-1)e^{-(x^2+y^2)}.$$

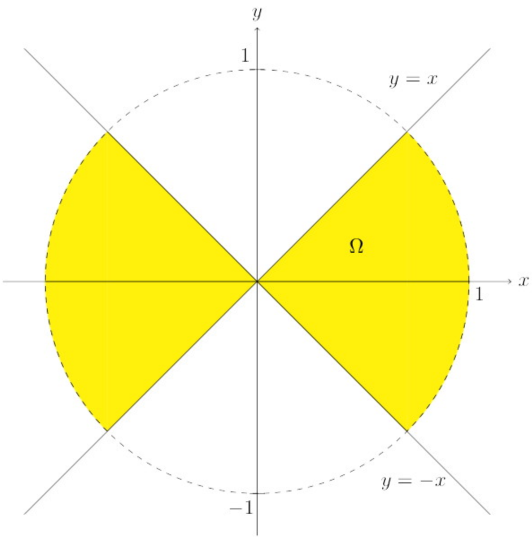
0.3. For which $a \in \mathbb{R}$ there exists a function $f : \mathbb{R}^3 \to \mathbb{R}$ whose gradient is the vector field

$$(x,y,z) \mapsto \begin{pmatrix} \log(1+x^2) + ay^2 \\ xy + y^2 \\ z^3 \end{pmatrix} ? \tag{1}$$

For such a's, find one function f. How do any such two such solutions (for the same a) differ?

0.4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) = \cos(x^2 + y^2)$$



Compute the Taylor polynomial of f of 3rd order at the origin.

0.5. Integrate the function

$$f(x, y) = |x|\sqrt{x^2 + y^2}$$

over the set Ω indicated below.

0.6. The Piriform curve is the planar curve given by

$$C := \{(x, y) \in \mathbb{R}^2 \mid y^2 = x^3(2 - x)\}.$$

A parametrization of C is given by $\gamma : [-\frac{\pi}{2}, \frac{3\pi}{2}] \to \mathbb{R}^2$,

$$\gamma(t) = \begin{pmatrix} 1 + \sin(t) \\ \cos(t)(1 + \sin(t)) \end{pmatrix}$$

Determine the area of the set Ω enclosed by C by means of Green's theorem.

0.7. Determine the values $\alpha, \beta \in \mathbb{R}$ for which integral in $\mathbb{R}^3$

$$\int_{\mathbb{R}^3} |x|^\alpha e^{-|x|^\beta} d\mu$$

is convergent (no need to compute it).

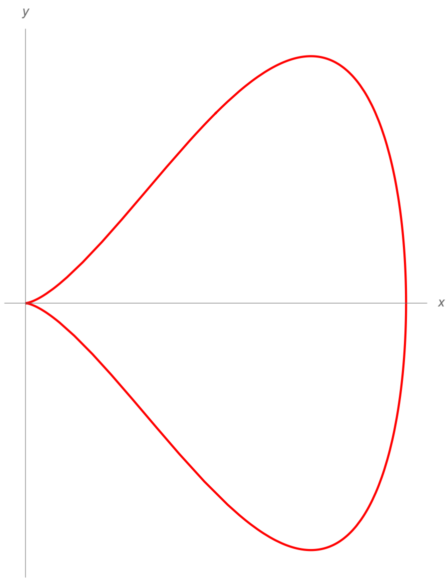


Figure 1: The Piriform Curve