Exercise 3 is taken from Dr. Menny Akka Ginosar's notes on Fibonnacci sequences. These notes are available on the course website.

1. The Golden Ratio Part 1. Recall that you computed $\mathcal{F}_{0,1}$ during the lectures. Now:

(a) Find a formula for $\mathcal{F}_{1,0}$.

(b) Write a formula for $\mathcal{F}_{a,b}$ that depends only on a, b and n.

2. Negation and De Morgan.

(a) Let A and B be statements. Show/explain the following:

(i) A is equivalent to $\neg(\neg A)$;

(ii) $\neg(A \vee B)$ is equivalent to $\neg A \wedge \neg B$;

(iii) $\neg(A \wedge B)$ is equivalent to $\neg A \vee \neg B$.

(b) Now, let A and B be two sets contained in a bigger set U. Show/explain the following:

(i) $(A \cup B)^c = A^c \cap B^c$;

(ii) $(A \cap B)^c = A^c \cup B^c$.

3. The Golden Ratio Part 2. We do not normally want to draw on your school knowledge apart from the skill of doing algebraic manipulations correctly. We make an exception in this task.

Let $\mathcal{F}_{0,1} = (F_0, F_1, F_2, \dots) \in \mathbf{Fib}$. Assuming that $\lim_{n\to\infty} \frac{F_n}{F_{n-1}}$ exists and isn't equal to 0, compute its value.

4. Logic. Let x, y, z be variables and let R, S be relations. Distribute the negation inside the parentheses in the following proposition:

$$\neg\Big(\forall x \exists y \exists z (R(x,y) \wedge R(z,x)) \vee (R(x,y) \wedge S(z,x))\Big)$$

5. Logic. Let $x, y \in \mathbb{R}$. Write the contrapositive of the following:

$$x \neq y \implies (\exists \epsilon > 0 : |x - y| > \epsilon)$$

6. Dimension. Let $\mathcal{F} = (\alpha_0, \alpha_1, \alpha_2, \dots)$, $\mathcal{G} = (\beta_0, \beta_1, \beta_2, \dots) \in$ **Fib**. You are asked to check whether $\mathcal{F} = \mathcal{G}$. The sequences themselves are not known but for any natural number i, you can check whether $\alpha_i = \beta_i$. You can check this for as many i's as you wish.

Claim: It is enough to check it for two **random** i's to know with certainty whether $\mathcal{F} = \mathcal{G}$, explain why.