EXCISION

A fundamental property of relative homology groups is given by the following **EXCISION THEOREM**, describing when the relative groups $H_n(X,A)$ are unaffected by excising/deleting a subset $Z \subset A$.

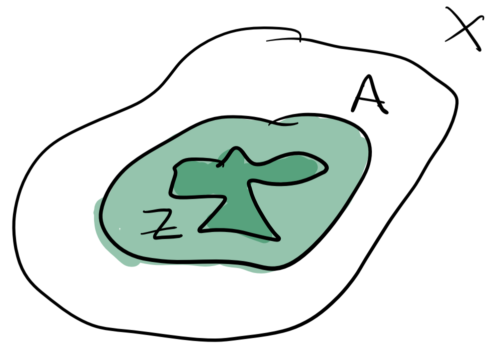
THEOREM (EXCISION)

Given subspaces $Z \subset A \subset X$ such that the closure of Z is contained in the interior of A, then the inclusion $(X-Z, A-Z) \hookrightarrow (X, A)$ induces isomorphisms $H_p(X-Z, A-Z) \to H_p(X,A)$ for all p.

Equivalently, for subspaces $A, B \subset X$ whose interior covers X, the inclusion $(B, A \cap B) \hookrightarrow (X, A)$ induces isomorphisms

$H_p(B, A\cap B) \to H_p(X, A)$ for all p.

The translation between the two versions is obtained by setting



$$B = X - Z \ \& \ Z = X - B.$$

Then $A \cap B = A - Z$ and the condition $\bar{Z} \subset \mathring{A}$ is equivalent to

$$X = \mathring{A} \cup \mathring{B} \text{ since } X - \mathring{B} = \bar{Z}.$$

The proof is quite technical and will be done in several steps.

RELATING HOMOLOGY GROUPS OF A COVERING TO HOMOLOGY GROUPS OF A SPACE

Let X be a space and $\mathcal{U} = \{U_\alpha\}_{\alpha \in A}$ be a collection of subsets of X s.t.