PROOF OF THEOREM 1  
Let 
$$U$$
 be a covering as in the  
statement of theorem 1. Let  $\partial \in Sp(x)$   
be a singular simplex.  
Then  $\{ \partial^{-1}(\hat{U}) \mid U \in U \}$  is  
an open covering of  $\Delta^{p}$ .  $\Delta^{p}$  is  
compact, so we can select the  
Lebesgue number 3 of this covering  
Pick  $m \in HI$  large enough that  
 $(p + A) \mid V \geq S$ .  
 $m$  will determine  
for subdivide simplices  
so that each lifes  
 $in$  some Uell

If we use sol m-times on

6 we get a chain consisting of singular simplices, of which each lies in some UEU.  $S_{c}(sd_{m}(id_{s})) = sd_{m}(S) \in S_{p}^{u}(x).$ For each p-simplex 2 we select Mg In a way that it is the smallest non-negative integr for which sdms (2)esp(x)  $(m\delta = 0 \leq 7\delta \in S_p^u(x)).$ We define  $\overline{D}: Sp(x) \rightarrow Sp_{1}(x)$  $\overline{D}(3) = \sum_{i=1}^{n} D(2q_i(3))$ Ú=0 - this is the D that for G a p-simplex we defined for singula chains

$$= \frac{i}{(i_{S})} \frac{i_{S}}{i_{S}} \frac{1}{i_{S}} \frac{i_{S}}{i_{S}} \frac{1}{i_{S}} \frac{i_{S}}{i_{S}} \frac{1}{i_{S}} \frac{1}{i_{S}}$$

We set  $p(2):=2-3\overline{D}(2)-\overline{D}(2)$ Note that  $p(2)\in S_p^u(x)$ . This  $p(2)\in S_p^u(x)$  or q or q or q or q.

$$\rho \text{ is a chain map:}$$

$$(5) \in GC - 3C = (5) =$$

$$= 3D - D3 = id - id^{u}p,$$
where  $i_{c}^{u} C_{n}^{u}(x) \rightarrow C_{n}(x)$  is
the inclusion.
$$D \text{ is a chain homotopy from}$$

$$i_{c}^{u}p \text{ to id.}$$
Also,  $po i_{c}^{u} p(i_{c}(c)) = \int_{0}^{0} p(i_{c}^{u}(c))$ 

$$= 3 - 3D(i_{c}^{u}(c)) - D(i_{c}^{u}(c))$$

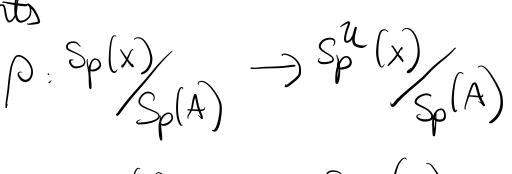
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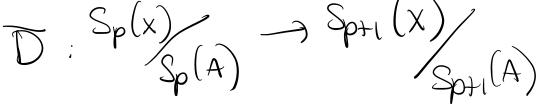
## so P is the chain homotopy inverse of $i_c^{\mathcal{U}}$ . It follows from homotopy invariance statements that $i_{\mathcal{X}}^{\mathcal{U}}$ is an isomorphism $H_p^{\mathcal{U}}(\mathbf{X}) \xrightarrow{i_{\mathcal{U}}^{\mathcal{U}}} H_p(\mathbf{X})$ .

PROOF OF EXCLOSION THEOREM Let U= JA, BY Such that AOB=X.  $\mathcal{L}_{\mathcal{L}}^{\mathcal{U}} : \mathcal{C}_{h}^{\mathcal{U}}(\mathbf{X}) \to \mathcal{C}_{h}(\mathbf{X})$ is a chain épuivalence. From proof of theorem I we get maps (> & D that map simplices in A to simplices in A.

## p and D induce maps on

guotients





If still holds that  $\partial \overline{D} + \overline{D} \partial = id - ic^{n} c^{n} c^{n}$ and that  $iu : C_{n}^{n} (x) \to C_{n} (x)$  $C_{n}(A) \to C_{n}(x)$ 

is a chain epuivalence and consequently it induces an isomorphism on homology.

 $S_{p}(B) \longrightarrow S_{p}^{u}(X)$  $S_{p}(A \cap B) \longrightarrow S_{p}^{u}(X)$ The map

Induced by inclusion is an isomorphism since both guotient groups are free with the basis Singular p-simplices in B that do not lie m A. =>  $H_{p}(x,A) \cong H_{p}(C_{n}(x)/C_{n}(A))$  $\cong \mathcal{H}_{p}(\mathcal{B},A\cap\mathcal{B})$ 

