Number theory I: Problem sheet 10

1. Find the integral solutions of the generalized Pell equation $x^2 - 15y^2 = \pm n$ for $n = 4, 7, 11$.

2. Find the integral solutions of the generalized Pell equation

$$x^2 - 10y^2 = n$$

   for $n = 7$ and 8.

3. Show that the regulator of a number field K as given in Definition 10.6 is well-defined.

4. Choose an ideal $\mathfrak{a} \in A^{-1}$. Then multiplication by $\mathfrak{a}$ gives a bijection between

$$\{\text{integral ideals in } A\} \quad \leftrightarrow \quad \{\text{principal ideals divisible by } \mathfrak{a}\}.$$

5. Show that the set X is definition 10.8 is a cone.