

## Number theory I: Problem sheet 6

1. Let  $p \geq 3$  be prime. Then

1.  $\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}};$

2.  $-1$  is a quadratic residue  $(\text{mod } p)$  if and only if  $p \equiv 1 \pmod{4}$ .

2. Let  $p$  be an odd prime. Show that  $3$  is a quadratic residue  $(\text{mod } p)$  if and only if  $p \equiv \pm 1 \pmod{12}$ .

3. Evaluate the following Legendre symbols:  $\left(\frac{107}{1009}\right), \left(\frac{21}{101}\right), \left(\frac{377}{233}\right), \left(\frac{-104}{131}\right)$ .

4. Show that if  $p$  is a prime of the form  $p = 2^n - 1$  for some  $n > 2$ , then  $\left(\frac{3}{p}\right) = 1$ .

5. Let  $p \equiv 3 \pmod{4}$  be prime, and let  $a$  be coprime to  $p$ . Show that either  $a$  or  $-a$  is a quadratic residue  $(\text{mod } p)$ , but not both.

6. Let  $p$  be an odd prime. Show that  $\sum_{a=1}^{p-1} \left(\frac{a}{p}\right) = 0$ .