Exercise sheet 2

1. Let G be a topological group.

   (a) Show that if $U \ni e$ is a neighborhood of the neutral element e, then there exists $V \ni e$ symmetric and open with $V^2 \subset U$.

   (b) Use (a) to show that if e is closed, then G is Hausdorff.

2. Let H be a topological group and $p\colon G \to H$ a covering map, where à priori G is just a topological space. Show that then for every choice $e_G \in p^{-1}(e_H)$ there exists a unique topological group structure on G with neutral element e_G such that $p\colon G \to H$ is a homomorphism.

3. Show that if $p\colon G \to H$ is a covering homomorphism of topological groups, then there is a local isomorphism from G to H and from H to G.

4. Show that the local homomorphism in the example at the beginning of Chapter 2.4 cannot be extended to a global continuous homomorphism.

5. Let G be a locally compact Hausdorff topological group and $\Lambda : C_{oo}(G) \to \mathbb{C}$ a positive linear functional that is represented by the regular Borel measure μ. Let $g \in G$. Show that the positive linear function $\lambda(g)^*\Lambda$ is represented by $g_*\mu$.

6. Define what a right (invariant) Haar functional on a locally compact group is. Prove that it always exists and that it is unique up to scalar multiple in $\mathbb{R}_{>0}$. Refer to page -2-48- in the notes.