Exercise Sheet 11

For each of the following multiple choice questions, choose **the** correct answer.

1. The radius of the osculating circle at a point of the helix $c(t) := (r\cos t, r\sin t, t)$ is:

(a) $\frac{r}{1+r^2}$.

(b) $\frac{1+r^2}{r}$.

(c) r.

(d) $\frac{1}{r}$.

(e) $\frac{\sqrt{1+r^2}}{r}$.

2. Let M be a smooth surface in $\mathbb{R}^3$ with Gauss curvature K and mean curvature H. Which of the following relations is always true?

(a) $H \geq K$.

(b) $H \leq K$.

(c) $H^2 \leq K$.

(d) $H^2 \geq K$.

(e) $H^2 = K^2$.

3. Consider the following curve in $\mathbb{R}^3$:

$$\gamma(t) = \big(3\cos(t/5), 4\cos(t/5), 5\sin(t/5)\big).$$

Which of the following vectors is the binormal B of γ?

(a) $(\frac{4}{5}, -\frac{3}{5}, 0)$.

(b) $(0, \frac{4}{5}, -\frac{3}{5})$.

(c) $(1, 0, 0)$.

(d) $(-\frac{3}{5}, \frac{4}{5})$.

(e) $\big(-\frac{3}{5}\sin(\frac{t}{5}), -\frac{4}{5}\sin(\frac{t}{5}), \cos(\frac{t}{5})\big)$.

4. Let $C \subset \mathbb{R}^3$ be the cylinder in $\mathbb{R}^3$, parametrized as

$$f(u, v) = \big(\cos(u), \sin(u), v\big).$$

What are the correct values of the Gauss curvature K and mean curvature H at the point $(\sqrt{2}/2, \sqrt{2}/2, 100) \in C$ (with respect to the outward pointing Gauss map) ?

(a) $K = 0, H = 0$.

(b) $K = 0, H = -\frac{1}{2}$.

(c) $K = -1, H = 0$.

(d) $K = 0, H = 1$.

(e) $K = 1, H = -1$.

5. Consider a “quadrilateral” region of area A in a 2-sphere of radius r (connected region bounded by four great circular arcs). The sum of its interior angles is:

(a) $2\pi - A/r^2$.

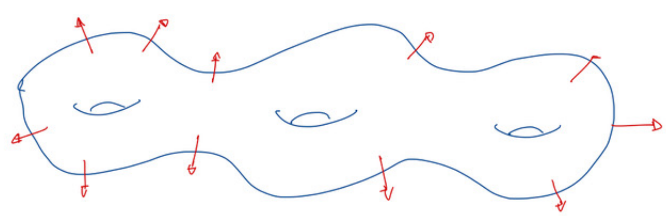
(b) $\pi - A$.

(c) $2\pi + A/r^2$.

(d) $2\pi r^2 + A$.

(e) $2\pi(1 + A/r^2)$.

6. Let $M \subset \mathbb{R}^3$ be the following smooth surface:



What is the degree of the outward pointing normal red vector field X?

(a) $\deg(X) = 0$.

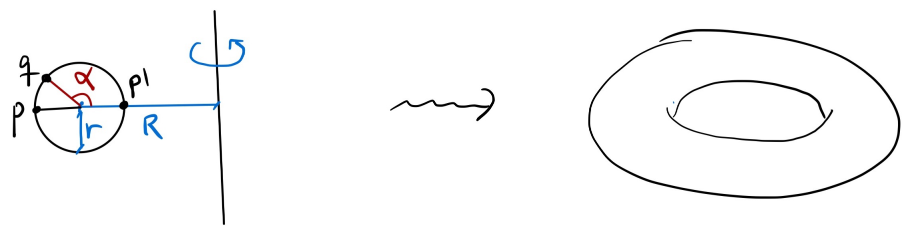
(b) $\deg(X) = 1$.

(c) $\deg(X) = 2$.

(d) $\deg(X) = -1$.

(e) $\deg(X) = -2$.

7. Consider the torus of revolution $f(x,y) = \big(\cos x(-R+r\cos y), \sin x(-R+r\cos y), r\sin y\big)$, $R > r$, drawn below:



Its Gauss' curvatures at $p = (-R-r,0,0)$ and $p' = (-R+r,0,0)$ are

(a) $\frac{1}{r\sqrt{R^2+r^2}}$ and $\frac{-1}{r\sqrt{R^2-r^2}}$, resp.

(b) $\frac{1}{rR}$ and $\frac{-1}{rR}$, resp.

(c) Both equal, in absolute value, to $\frac{1}{\sqrt{rR}}$.

(d) Both equal, in absolute value, to $\frac{1}{rR}$.

(e) $\frac{1}{r(R+r)}$ and $\frac{-1}{r(R-r)}$, resp.

8. Consider again the torus from question 7. The mean curvature at the point $q = (-R + r\cos\alpha, 0, r\sin\alpha)$ with respect to the outwards normal (pointing towards the unbounded component of $\mathbb{R}^3 \setminus f([0,2\pi]^2)$) is:

(a) $\frac{1}{2}\big(-\frac{1}{r} + \frac{1}{R-r\cos\alpha}\big)$.

(b) $\frac{1}{2}\big(-\frac{1}{r} + \frac{\sin\alpha}{R-r\cos\alpha}\big)$.

(c) $\frac{1}{2}\big(-\frac{1}{r} + \frac{\tan\alpha}{R-r}\big)$.

(d) $\frac{1}{2}\big(-\frac{1}{r} + \frac{\cos\alpha}{R-r\cos\alpha}\big)$.

(e) $\frac{1}{2}\big(-\frac{1}{r} + \frac{\tan\alpha}{R+r}\big)$.

9. Consinder again the torus from question 7. When the point q is rotated about the x_3 axis it generates the curve $\gamma(t) = (\cos t(-R + r\cos\alpha), \sin t(-R + r\cos\alpha), r\sin\alpha)$, which is contained in the torus. Given a tangent vector X at q consider its parallel transport along γ for one full turn ($t \in [0, 2\pi]$), producing a new tangent vector Y at q. The angle between X and Y is:

(a) $\frac{\alpha R}{r}$.

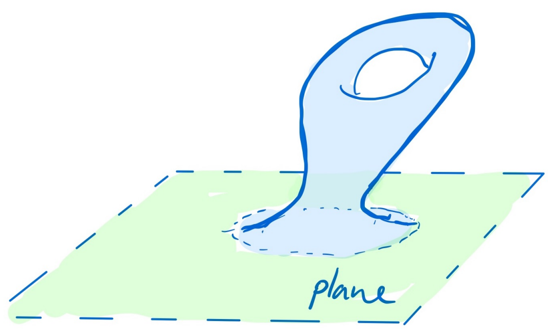
(b) $2\pi \sin\alpha$.

(c) $\frac{\tan\alpha R}{r}$.

(d) $2\pi \cos\alpha$.

(e) $\sin\alpha$.

10. Consider a smooth surface S obtained by gluing a torus (minus a disk) and a rectangular piece of plane (minus a disk), as in the figure. While the torus part was stretched in order to be tangent to the plane, the planar part was kept exactly flat.



Then $\int_S K dA$ is

(a) 4π.

(b) -4π.

(c) 2π.

(d) -2π.

(e) It depends on the curve bounding the planar piece of surface