Non-Life Insurance: Mathematics and Statistics

Exercise sheet 3

Exercise 3.1 No-Claims Bonus
An insurance company decides to offer a no-claims bonus to good car drivers, namely

- a 10% discount on the premium after three years of no claim, and
- a 20% discount on the premium after six years of no claim.

How does the premium need to be adjusted such that the premium income is expected to remain the same as before the grant of the no-claims bonus? For simplicity, we consider one car driver who has been insured for at least six years. Answer the question in the following two situations:

(a) The claim counts of the individual years of the considered car driver are i.i.d. Poisson distributed random variables with frequency parameter $\lambda = 0.2$.

(b) Suppose Θ follows an exponential distribution with parameter $c = 1$. Conditionally given Θ, the claim counts of the individual years of the considered car driver are i.i.d. Poisson distributed random variables with frequency parameter $\Theta\lambda$, where $\lambda = 0.2$ as above.

Exercise 3.2 Compound Poisson Distribution
For the total claim amount S of an insurance company we assume $S \sim \text{CompPoi}(\lambda v, G)$, where $\lambda = 0.06$, $v = 10$ and for a random variable Y with distribution function G we have

k	100	300	500	6'000	100'000	500'000	2'000'000	5'000'000	10'000'000
$\mathbb{P}[Y = k]$	3/20	4/20	3/20	2/15	2/15	1/15	1/12	1/24	1/24

Table 1: Claim size distribution $Y \sim G$.

Suppose that the insurance company wants to distinguish between

- small claims: claim size $\leq$ 1'000,
- medium claims: 1'000 $<$ claim size $\leq$ 1'000'000 and
- large claims: claim size $>$ 1'000'000.

Let S_{sc}, S_{mc} and S_{lc} be the total claim in the small claims layer, in the medium claims layer and in the large claims layer, respectively.

(a) Give definitions of S_{sc}, S_{mc} and S_{lc} in terms of mathematical formulas.

(b) Determine the distributions of S_{sc}, S_{mc} and S_{lc}.

(c) What is the dependence structure between S_{sc}, S_{mc} and S_{lc}?

(d) Calculate $\mathbb{E}[S_{\text{sc}}]$, $\mathbb{E}[S_{\text{mc}}]$ and $\mathbb{E}[S_{\text{lc}}]$ as well as $\text{Var}(S_{\text{sc}})$, $\text{Var}(S_{\text{mc}})$ and $\text{Var}(S_{\text{lc}})$. Use these values to calculate $\mathbb{E}[S]$ and $\sqrt{\text{Var}(S)}$.

(e) Calculate the probability that the total claim in the large claims layer exceeds 5 million.

Exercise 3.3 Compound Distribution
Assume that

$$S = \sum_{i=1}^{N} Y_i$$

has a compound distribution with N having a geometric distribution with parameter $p \in (0,1)$ and $Y_1, Y_2, \ldots$ being i.i.d. exponentially distributed with parameter $\lambda > 0$. Show that S is exponentially distributed with parameter λp.

Exercise 3.4 Compound Binomial Distribution
Assume $S \sim \mathrm{CompBinom}(v, p, G)$ for given $v \in \mathbb{N}$, $p \in (0,1)$ and individual claim size distribution G. Let $M > 0$ such that $G(M) \in (0,1)$. Define the compound distribution S_{sc} with individual claims Y_i that are at most of size M and the compound distribution S_{lc} with individual claims Y_i that exceed threshold M by, respectively,

$$S_{\mathrm{sc}} = \sum_{i=1}^{N} Y_i \, 1_{\{Y_i \le M\}} \qquad \text{and} \qquad S_{\mathrm{lc}} = \sum_{i=1}^{N} Y_i \, 1_{\{Y_i > M\}}.$$

(a) Show that $S_{\mathrm{lc}} \sim \mathrm{CompBinom}(v, p[1 - G(M)], G_{\mathrm{lc}})$, where the large claims size distribution function G_{lc} satisfies $G_{\mathrm{lc}}(y) = \mathbb{P}[Y_1 \le y | Y_1 > M]$.

(b) Give a short argument which shows that S_{sc} and S_{lc} are not independent.