Non-Life Insurance: Mathematics and Statistics

Exercise sheet 4

Exercise 4.1 Poisson Model and Negative-Binomial Model

Suppose that we are given the following claim count data of ten years:

ſ	t	1	2	3	4	5	6	7	8	9	10
ſ	N_t	1'000	997	985	989	1'056	1'070	994	986	1'093	1'054
	v_t	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000	10'000

Table 1: Observed claim counts N_t and corresponding volumes v_t .

- (a) Estimate the claim frequency parameter $\lambda > 0$ of the Poisson model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies N_t/v_t . What do you observe?
- (b) Perform a χ^2 -goodness-of-fit test at significance level of 5% to test the null hypothesis of having Poisson distributions.
- (c) Estimate the claim frequency parameter $\lambda > 0$ and the dispersion parameter $\gamma > 0$ of the negative-binomial model. Moreover, calculate a prediction interval which should contain roughly 70% of the observed claim frequencies N_t/v_t . What do you observe?

Exercise 4.2 χ^2 -Goodness-of-Fit-Analysis (R Exercise)

In this exercise we analyze the sensitivity of the χ^2 -goodness-of-fit test (of having a Poisson distribution as claim count distribution) in situations where the claim counts are simulated from a Poisson distribution and a negative binomial distribution, respectively.

- (a) Write an R code that generates n=10'000 times claim counts $N_1, \ldots, N_T \stackrel{\text{i.i.d.}}{\sim} \text{Poi}(\lambda v)$ with $T=10, \ \lambda=10\%$ and v=10'000. Apply for each of these n replications of N_1, \ldots, N_T a χ^2 -goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:
 - (i) What can you say about the distribution of the n MLEs of λ ?
 - (ii) Consider a QQ plot to analyze whether the n values of the test statistic may indeed come from a χ^2 -distribution with T-1=9 degrees of freedom.
 - (iii) How often do we wrongly reject the null hypothesis H_0 of having a Poisson distribution as claim count distribution?
- (b) Write an R code that generates n=10'000 times claim counts $N_1,\ldots,N_T \overset{\text{i.i.d.}}{\sim} \text{NegBin}(\lambda v,\gamma)$ with $T=10,\,\lambda=10\%,\,v=10'000$ and $\gamma\in\{100,1'000,10'000\}$. Apply for each of these n replications of N_1,\ldots,N_T a χ^2 -goodness-of-fit test at significance level of 5% of having a Poisson distribution as claim count distribution. Answer the following questions:
 - (i) How often are we able to reject the null hypothesis H_0 of having a Poisson distribution as claim count distribution?
 - (ii) Does the size of γ influence this percentage?

Exercise 4.3 Claim Count Distribution

Suppose that in a given line of business of an insurance company the numbers of claims of the last ten years are modeled by random variables N_1, \ldots, N_{10} . We assume that N_1, \ldots, N_{10} are i.i.d. and that we have collected the following observations:

	t	1	2	3	4	5	6	7	8	9	10
Ì	N_t	7	21	19	18	25	17	33	6	39	28

Table 2: Observed numbers of claims N_t over the last ten years.

Which claim count distribution would you prefer in this situation? Give a short argument.

Exercise 4.4 Method of Moments

The i.i.d. claim sizes Y_1, \ldots, Y_8 are supposed to follow a Gamma distribution with unknown shape parameter $\gamma > 0$ and unknown scale parameter c > 0. We assume that we have the following observations for Y_1, \ldots, Y_8 :

$$y_1 = 7$$
, $y_2 = 8$, $y_3 = 10$, $y_4 = 9$, $y_5 = 5$, $y_6 = 11$, $y_7 = 6$, $y_8 = 8$.

Use the method of moments to estimate the parameters γ and c.