ETH Zürich, HS 2022 — Coordinator
Prof. Vincent Tassion — Daniel Contreras

RANDOM WALKS ON TRANSITIVE GRAPHS (D-MATH)
EXERCISE SHEET 1

Definition: We say that a graph $G = (V, E)$ is *transitive* (or *vertex transitive*) if for every $x, y \in V$ there exists $\varphi \in \mathrm{Aut}(G)$ such that $\varphi(x) = y$.

($\star$) **Exercise 1.** Let us consider the graph $G = (V, E)$, with

$$V = \bigcup_{n\in\mathbb{Z}} \{n\} \times 2^n\mathbb{Z}$$

$$E = \big\{\{(n, k \cdot 2^n), (n-1, 2k \cdot 2^{n-1})\}, n \in \mathbb{Z}, k \in \mathbb{Z}\big\}$$
$$\cup \big\{\{(n, k \cdot 2^n), (n-1, (2k+1) \cdot 2^{n-1})\}, n \in \mathbb{Z}, k \in \mathbb{Z}\big\}.$$

(i) Make a drawing of G. Do you recognise this graph?
(ii) Show that G is transitive.

($\star$) **Exercise 2.** Let $G = (V, E)$ be a locally finite transitive graph.

(i) Prove that for every $x, y \in V$, $\deg(x) = \deg(y)$.
(ii) Let $\varphi \in \mathrm{Aut}(G)$. Prove that for every $x, y \in V$, $d(\varphi(x), \varphi(y)) = d(x, y)$, where d denotes the graph distance.

Exercise 3. Let $G = (V, E)$ be a graph. We say that G is *edge transitive* if for every $e_1, e_2 \in E$, there is an automorphism of G that maps e_1 to e_2.

(i) Show that if G is edge transitive then it is not necessarily vertex transitive.
(ii) Show that the reciprocal is also not true.