Mathematical Tools I 2023

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Self Assessment

Please note that this self assessment is NOT graded!
We recommend that you solve it under open-book exam conditions and limit your time to 70 - 80 minutes. Note for some exercises more than one answer is correct! Please submit your solutions by the 12th of January at noon.

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1. Consider the matrix $E = \begin{pmatrix} 0 & -1 & 0 \\ b & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ for a constant $b \in \mathbb{R}$. Let $F = \begin{pmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a second matrix. Then the product EF is given by ...

(a) $\begin{pmatrix} -b & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(c) $\begin{pmatrix} -2 & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(d) $\begin{pmatrix} b & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

2. Consider again the matrices $E = \begin{pmatrix} 0 & -1 & 0 \\ b & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $F = \begin{pmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. If $b > 0$ then ...

(a) the eigenvalues of the product EF are negative numbers.

(b) the sum of the eigenvalues of the product EF is a negative number.

(c) the product of the eigenvalues of the product EF is a negative number.

(d) None of the above.

3. The matrix $A = \begin{pmatrix} 5 & 1 \\ -4 & 1 \end{pmatrix}$ has eigenvalues $\alpha_1 = 3$ and $\alpha_2 = \ldots$

(a) $\alpha_2 = 1$.

(b) $\alpha_2 = 2$.

(c) $\alpha_2 = 3$.

(d) $\alpha_2 = 4$.

4. Which of the following vectors is an eigenvector of $\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(d) None of the above.

5. Let $A = \begin{pmatrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} x_1 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 1 & x_2 \end{pmatrix}$. Which x_1, x_2 yield $B = A^{-1}$?

(a) $x_1 = 1$ and $x_2 = -1$

(b) $x_1 = -1$ and $x_2 = 1$

(c) $x_1 = x_2 = 1$

(d) $x_1 = x_2 = -1$

6. The inverse of $A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \dots$

(a) exists only if $\varphi = 0, \pi$.

(b) is given by $A^{-1} = \dfrac{1}{\cos^2\varphi - \sin^2\varphi} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$.

(c) is given by $A^{-1} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$.

(d) is given by $A^{-1} = \begin{pmatrix} \sin\varphi & -\cos\varphi \\ \cos\varphi & \sin\varphi \end{pmatrix}$.

7. Let $A = \begin{pmatrix} 0 & -4 & -3 \\ 2 & 2 & 2 \\ 1 & 4 & 4 \end{pmatrix}$. The vector $v = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ is an eigenvector of the **inverse** matrix A^{-1} with eigenvalue λ. Find the value of λ.

(a) $\lambda = \frac{1}{2}$

(b) $\lambda = 2$

(c) $\lambda = \frac{1}{3}$

(d) $\lambda = 3$

8. Which x yields $\det \begin{pmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix} = 1$?

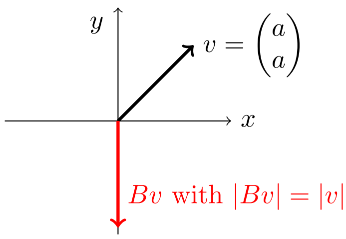
(a) $x = 0$

(b) $x = 1$

(c) $x = 2$

(d) $x = 4$

9. Let $B = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix}$ and $v = \begin{pmatrix} a \\ a \end{pmatrix}$ with $a > 0$. Which φ with $0 \leq \varphi < 2\pi$ matches the following figure?



(a) $\varphi = \frac{\pi}{4}$

(b) $\varphi = \frac{3\pi}{4}$

(c) $\varphi = \frac{5\pi}{4}$

(d) $\varphi = \frac{7\pi}{4}$

10. Which of the following sets is a subspace of the vector space of (2×2) - matrices $M_{2\times 2}$ with the usual addition and scalar multiplication?

(a) The set of all invertible (2×2) - matrices.

(b) $U = \left\{ \begin{pmatrix} a & 0 \\ a^2 & a \end{pmatrix} ; a \in \mathbb{R} \right\}$

(c) $U = \left\{ A \in M_{2\times 2} : A^\top = -A \right\}$

(d) None of the above.

11. The imaginary part of $z = \dfrac{8}{i+1}$ is given by ...

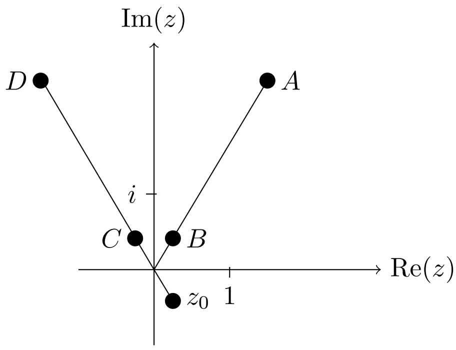
(a) $\Im(z) = -8.$

(b) $\Im(z) = -4.$

(c) $\Im(z) = 4.$

(d) $\Im(z) = 8.$

12. Let z_0 and A, B, C, D be points representing complex numbers as indicated below



Which point represents $\dfrac{1}{z_0}$?

(a) A

(b) B

(c) C

(d) D

13. Let $\begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ -1 & 0 & b \end{pmatrix} \cdot x = 0$. For which b does this equation have a solution $x \neq 0$?

(a) $b = -2$

(b) $b = -1$

(c) $b = 0$

(d) $b = 1$

14. The above matrix is involved in the following equation $\begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 0 \\ -1 & 0 & b \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ z \end{pmatrix}$. There exist b and z such that $\begin{pmatrix} 0 \\ 1 \\ \frac{1}{4} \end{pmatrix}$ and $\begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}$ are both solutions. Find the value of z.

(a) $z = -2$

(b) $z = -\frac{1}{2}$

(c) $z = \frac{1}{2}$

(d) $z = 1$

15. Consider the following matrix in row reduced echelon form:

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & \alpha+1 & 0 & \beta^2-1 \\ 0 & 0 & \alpha-2 & \beta+1 \end{array}\right]$$

For which choice of α and β does this system have a unique solution?

(a) $\alpha = 2$ and any β

(b) $\alpha = 2$ and $\beta = -1$

(c) $\alpha \neq 2$ and any β

(d) None of the above.

16. Which of the following ODEs are linear?

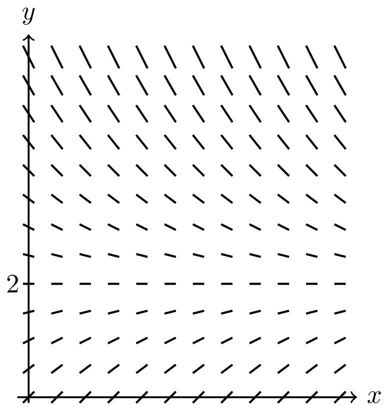
(a) $(y' - 2)^2 = y$

(b) $\dfrac{y'}{1-x^2} + \dfrac{y}{1+x} = \dfrac{1}{x^2}$

(c) $y' = \dfrac{2xy}{x^2 - y^2}$

(d) None of the above.

17. Consider the following slope field.



Which of the following ODEs fits?

(a) $y'(x) = \frac{1}{2} \cdot y(x) + 1$

(b) $y'(x) = 2y(x) + 1$

(c) $y'(x) = -\frac{1}{2} \cdot y(x) + 1$

(d) None of the above.

18. Which general solution matches the above slope field?

(a) $y(x) = C \cdot e^{\frac{1}{2}x} + 1$

(b) $y(x) = C \cdot e^{-\frac{1}{2}x} - 2$

(c) $y(x) = C \cdot e^{\frac{1}{2}x} + 2$

(d) $y(x) = C \cdot e^{-\frac{1}{2}x} + 2$

19. Which of the $y(x)$ defines the general solution of $y'' - \omega^2 y = 0$ (with $\omega \neq 0$ constant)?

(a) $y(x) = C_1 e^{\omega x} + C_2 \cdot x \cdot e^{-\omega x}$

(b) $y(x) = C_1 e^{\omega x} + C_2 e^{-\omega x}$

(c) $y(x) = C_1 e^{\omega^2 x} + C_2 e^{-\omega^2 x}$

(d) None of the above.

20. What is the extension of MATLAB script files?

(a) `.mat`

(b) `.m`

(c) `.script`

(d) There are no script files in MATLAB.

21. The `if else if` statement is used for:

(a) one statement

(b) two variables

(c) multiple statements

(d) none of the above.

22. The MATLAB expression `rand` generates a real number in the open interval $(0,\ 1)$. Which of the following would generate a random number in the open interval $(3,\ 5)$?

(a) `rand*2+3`

(b) `rand*3+5`

(c) `rand*[3,5]`

23. Which statement returns the roots for the polynomial $x^2 - x + 3$?

(a) `poly([1 -1 3])`

(b) `solve(x^2-x+3==0)`

(c) `polyfit(x^2-x+3==0)`

(d) `roots([1 -1 3])`

24. How many times will the following loop run?

```
for i=1:10
if (i < 4) break
end
end
```

(a) It will result in an error.

(b) 0 times

(c) 4 times

(d) 10 times