These closed-answer questions cover some topics from previous classes that will be useful for this course. If you find some of them obscure you are encouraged to revise briefly the relative topic and/or to come to office hours.

0.1. Inclusion between L^p spaces.

1. Is it true that if $f \in L^1(\mathbb{R}^n)$ then necessarily $f \in L^2(\mathbb{R}^n)$?
2. Is it true that if $f \in \ell^1(\mathbb{N})$ then necessarily $f \in \ell^2(\mathbb{N})$?
3. $L^2(0,1) \subsetneq L^1(0,1)$?

0.2. Completeness and Cauchy sequences.

1. Is it true that a Cauchy sequence (say, in a metric space) can have at most one limit?
2. Is it true that the interval $(0,1) \subset \mathbb{R}$ is complete?
3. Is L^2 complete with respect to the L^1 distance? That is to say: a sequence of L^2 functions $\{f_k\}$ which is Cauchy with respect to the $\|\cdot\|_{L^1}$ norm needs to converge (in the L^1 norm) to some $f \in L^2$?
4. Can you build a sequence of functions $\{f_k\} \subset L^2(\mathbb{R})$ such that

$$\int_{\mathbb{R}} |f_k(x) - 1|^2 \, dx \to 0 \text{ as } k \to \infty\,?$$

0.3. Linear algebra and the spectral Theorem.

1. Is it true that a self-adjoint matrix with complex entries admits a basis of eigenvectors, which are pairwise orthogonal and correspond to real eigenvalues?
2. Is it true that step functions form a vector subspace of $L^1(0,1)$? Is it dense?
3. How many dense vector subspaces there are in $\mathbb{C}^d$?
4. Is it true that any vector space admits an algebraic basis?