

## Exercise Sheet 5

### Exercise 1

We consider the Cayley-transformation  $r_2: \hat{\mathbb{C}} \rightarrow \hat{\mathbb{C}}, z \mapsto \frac{z-i}{z+i}$ .

- (a) Verify that  $r_2$  sends the real line to  $S^1$ .
- (b) Where does  $r_2$  send  $S^1$ ?
- (c) Draw a picture of how  $r_2$  acts on the Riemann sphere  $\hat{\mathbb{C}}$  viewed as a sphere  $S^2 \subseteq \mathbb{R}^3$ .
- (d) Find an explicit formula for  $r_2^{-1}$ .

### Exercise 2

Show that the subgroup  $\mathrm{PSL}(2, \mathbb{R})$  of the orientation preserving Möbius transformations  $\mathrm{PSL}(2, \mathbb{C}) \cong \mathrm{Möb}_+$  preserves the upper half plane  $H$ .

### Exercise 3

Let  $p \in B \setminus \{0\}$  be a point in the unit disk  $B$ . Construct the image of  $p$  under inversion in the unit circle using only compass and straightedge.

*Hint: Draw two straight lines through  $p$  and figure out what the circle inversion does to the two lines.*

### Exercise 4

Consider the orientation-preserving octahedral group  $O$  as a subgroup of  $\mathrm{Isom}_+(S^2)$ .

- (a) How many elements does it have?
- (b) List all elements by their order.