Exercise Sheet 6

Exercise 1

(a) Show that Möb_+ acts transitively on the set of clines, i.e. for any two clines ℓ, ℓ' there is $g \in \text{Möb}_+$ with $g\ell = \ell'$.

(b) For $z \in \hat{\mathbb{C}}$, what is the point $z' \in \hat{\mathbb{C}}$ that lies on the opposite side of the Riemann sphere $S^2 \cong \hat{\mathbb{C}}$?

(c) What are the clines in $\hat{\mathbb{C}}$ that correspond to great circles[1] on S^2? Which great circles correspond to lines? For those great circles that correspond to circles, how do their radii depend on their centers?

Exercise 2

Let $t \in \mathbb{R}$ and

$$U_t(z) = \frac{z}{1 + tz}$$

(a) Consider the family of all clines tangent to the imaginary axis at the origin. Prove that U_t takes each member of this family to another one.

Hint: compute $U_t'(0)$ and consider its effect.

(b) Consider the family of all circles tangent to the real axis at the origin. Prove that U_t preserves each member of this family but slides it along itself (as t varies).

(c) Show these families of circles are perpendicular to one another wherever they meet.

(d) Draw these families. Draw arrows to indicate the motion effected by U_t.

Exercise 3

Show that the set of all inversions in clines generate Möb.

[1] A *great circle* on S^2 is a circle that contains two opposite points