# Exercise Sheet 8

## Exercise 1

(a) Find an example of an elliptic Möbius transformation that preserves the unit disk.

(b) Find an example of a hyperbolic Möbius transformation that preserves the unit disk.

(c) Find an example of a parabolic Möbius transformation that preserves the unit disk.

(d) Show that there is no loxodromic Möbius transformation that preserves the unit disk, except for the hyperbolic or elliptic ones.

## Exercise 2

(a) Find the stabilizer[1] of 0 in $\mathrm{Möb}(B_1)$.

(b) For a point $x \in B_1$, describe the stabilizer of $x$ in $\mathrm{Möb}(B_1)$ in terms of the group found in (a).

(c) How many elements of $\mathrm{Möb}(B_1)$ fix two points $x \neq y \in B_1$?

## Exercise 3

Let $0 < t < 1$ and $K_t$ the Apollonian slide defined by

$$K_t(z) = \frac{z+t}{tz+1}.$$

(a) Show that for all $s \in (-1, 1)$, $K_t(s) > s$.

(b) Show that for all $z \in B_1$, $\lim_{n\to\infty} K_t^n(z) = 1$ and $\lim_{n\to-\infty} K_t^n(z) = -1$. *Hint: It may help to see $K_t$ as a one-parameter subgroup as in Sheet 7, Exercise 1.*

[1] The *stabilizer* of $x \in X$ in a group $G$ acting on $X$ is the subgroup $\{g \in G \colon g(x) = g\}$.