ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Probability and Statistics
D-MATH
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Sheet 1

Due: To be handed in before 03.03.2023 at 12:00.

1. Exercise

Let A, B, C be three events. Express the following events using A, B, C and the operations $\cap$, $\cup$, ^{c} (complement of a set).

$E_1 = $ "At least one of the events A, B, C occurs."

$E_2 = $ "At most one of the events A, B, C occurs."

$E_3 = $ "Exactly one of the events A, B, C occurs."

$E_4 = $ "B can only occur when A or C occurs."

$E_5 = $ "If A occurs, then B occurs as well."

$E_6 = $ "At least one of the events A, B, C occurs, but not all of them at the same time."

2. Exercise

We throw a green and a red die and consider the following events.

$E_1 = $ "None of the numbers is bigger than 2."

$E_2 = $ "The numbers are equal."

$E_3 = $ "The number on the red die is twice the number on the green die."

$E_4 = $ "The number on the red die is exactly one smaller or one bigger than the number on the green die."

$E_5 = $ "If the number on the red die is at most 5, then the number on the green die is equal to 6."

(a) Write down the sample space Ω for this random experiment and express the above events as subsets of Ω.

(b) Which one of the above events remains unchanged if we do not know anymore the color of the dice?

3. Exercise

Consider an urn containing N numbered balls, K of which are red and $N - K$ are white. Without loss of generality, we may assume that the balls with numbers $1, 2, \ldots, K$ are red. Now we draw <u>without</u> replacement n balls from the urn ($n \leq N$).

(a) What is the sample space Ω that corresponds to this random experiment?

(b) Find the cardinality $|\Omega|$ of Ω.

(c) Consider the event $R_k = \{$there are exactly k red balls in the sample$\}$. Find the cardinality $|R_k|$ for every $k \in \{0, \ldots, K\}$.

(d) Compute the ratio $|R_k|/|\Omega|$. Assuming that $K, N \to \infty$ and $K/N \to p$ for a $p \in (0, 1)$, to which limit does $|R_k|/|\Omega|$ converge?