ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Probability and Statistics
D-MATH
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Sheet 10

Due: To be handed in before 12.05.2023 at 12:00.

1. Exercise

We observe i.i.d. random variables $X_1, \ldots, X_n$ from a Poisson distribution with an unknown intensity $\lambda \in \Theta = (0, \infty)$. We want to test

$$H_0 \colon \lambda = \lambda_0 \qquad \text{versus} \qquad H_1 \colon \lambda = \lambda_1 \tag{1}$$

for some $\lambda_1 > \lambda_0$. We choose $\alpha = 0.05$.

(a) Recall the density of $X \sim \text{Pois}(\lambda)$ (with respect to the counting measure).

(b) Recall the Neyman-Pearson test for simple hypotheses at level α.

(c) Find the NP-test for the testing problem in (1) at level α.

(d) Using the result that $\sum_{i=1}^{n} X_i \sim \text{Pois}\left(\sum_{i=1}^{n} \lambda_i\right)$ if $X_i \sim \text{Pois}(\lambda_i)$ and $X_1, \ldots, X_n$ are independent, give the NP-test more explicitly if you know that $n = 30$, $\lambda_0 = 40$, and if $Y \sim \text{Pois}(1200)$.

x	$\mathbb{P}(Y \leq x)$
1254	0.9413
1255	0.9446
1256	0.9477
1257	0.9506
1258	0.9535
1259	0.9562
1260	0.9587

(e) The manager of a bakery would like to employ an additional person for the morning shift (7a.m. – 11a.m.). To make a case, he needs to prove that the rate at which the customers come during this period is equal to 45. Based on data collected in the past, this rate was until now assumed to be 40. During the month of April, the manager counted the number of customers. He got the following data:

$47, 43, 50, 48, 44, 58, 43, 39, 60, 48, 51, 35, 47, 48, 45, 49, 48, 34, 49, 40, 42, 37, 40, 51, 53, 33, 45, 43, 43, 39.$

Can the manager build their case for employing an additional person?

2. Exercise

Let X be a random variable such that $X(\Omega) = \mathcal{X} = \{1, 2, 3, 4, 5, 6, 7\}$. Under some null and alternative hypotheses H_0 and H_1, we know that the probability mass function f of X is f_0 and f_1, respectively, and

x	1	2	3	4	5	6	7
$f_0(x)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f_1(x)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

We want to test

$$H_0 \colon f = f_0 \qquad \text{versus} \qquad H_1 \colon f = f_1.$$

We choose $\alpha = 0.04$.

(a) What is the NP-test of level α for this testing problem?

(b) Compute the power of the NP-test found in (a).