ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Probability and Statistics
D-MATH
Prof. Dr. F. Balabdaoui

Sheet 6

Due: To be handed in before 07.04.2023 at 12:00.

1. Exercise

Consider the following joint probability density of some random pair (X, Y) for some $c > 0$;

$$f(x, y) = \begin{cases} cxy & \text{if } 1 \leq x \leq 3, \ 1 \leq y \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find c.

(b) Are the random variables X and Y independent?

(c) Compute $\mathbb{E}[X]$, $\mathbb{E}[Y]$ and $\mathbb{E}[XY]$.

2. Exercise

Let f be the following joint probability density of some random pair (X, Y) for some $\alpha > 0$;

$$f(x, y) = \alpha \frac{1}{x^2} \mathbb{1}_{1 \leq x \leq y} \mathbb{1}_{[1,2]}(y).$$

(a) Find α.

(b) Are the random variables X and Y independent?

(c) Compute the covariance $\text{cov}(X, Y)$ and the correlation $\rho(X, Y)$.

3. Exercise

Let X and Y be two i.i.d. $\sim \mathcal{G}(p)$ geometric random variables for some $p \in (0, 1)$. Compute $\mathbb{P}(X \geq Y)$ and $\mathbb{P}(X > Y)$.