Sheet 9

Due: To be handed in before 05.05.2023 at 12:00.

1. Exercise

Let $X_1, \dots, X_n$ be i.i.d. random variables $\sim$ Bernoulli(θ_0) for some unknown $\theta_0 \in \Theta = (0, 1)$.

(a) Show that the moment estimator of θ_0 is $\bar{X}_n$.

(b) Using the Central Limit Theorem, state the asymptotic distribution of $\hat{\theta}$.

(c) Using the weak law of large numbers and Slutsky's theorem, find a bilateral and symmetric confidence interval for θ_0 of asymptotic level equal to 95%.

2. Exercise

Let $X_1, \dots, X_n$ be i.i.d. $\sim \mathcal{U}(0, \theta_0)$ for some $\theta_0 \in \Theta = (0, \infty)$. For $X \sim \mathcal{U}(0, \theta_0)$, we adopt the following expression for the density: $p_\theta(x) = \frac{1}{\theta} \mathbb{1}_{0 \le x \le \theta}$.

(a) Write the likelihood based on the sample $\mathbb{X} = (X_1, \dots, X_n)$ and show that the MLE is $\hat{\theta} = \max_{1 \le i \le n} X_i$.

(b) Show that $\hat{\theta} \xrightarrow{\mathbb{P}} \theta_0$.

Hint: We can first show that $\mathbb{E}_{\theta_0}[|\hat{\theta} - \theta_0|] \to 0$ as $n \to \infty$.