Exercise Sheet 2

Algebraic Topology II

27.03.2023

Q1 Let X be a topological space. Show that $H^1(X, \mathbb{Z})$ has no torsion.

Q2 Compute the singular cohomology groups with $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients of the following spaces via simplicial or cellular cohomology and check the universal coefficient theorem in this case.

1. The two dimensional torus $T = S^1 \times S^1$.
2. The Klein bottle.
3. The real projective plane $\mathbb{R}P^2$.

Q3 Show that if $f : S^n \to S^n$ $(n \geq 1)$ had degree d then $f^* : H^n(S^n; G) \to H^n(S^n; G)$ is given by multiplication by d.

Q4 Show that $\mathrm{Tor}(A, \mathbb{Q}/\mathbb{Z})$ is isomorphic to the tosrion subgroup of A. Deduce that A is torsion-free if and only if $\mathrm{Tor}(A, B) = 0$ for all abelian group B.

Q5 Show that $\mathrm{Tor}(A, B)$ is always a torsion group.

Q6 In the example of $\mathbb{R}P^2$ and the coefficient ring $\mathbb{Z}_2$ check the statement of the universal coefficient theorem in homology.