

# Exercise Sheet 4

## Algebraic Topology II

15.05.2023

**Q1** Let  $M, N$  be topological manifolds. Show that  $M \times N$  is orientable if and only if  $M$  and  $N$  are both orientable.

**Q2** Show that every covering space of an orientable manifold is an orientable manifold.

**Q3** Show that for any connected closed orientable  $n$ -manifold  $M$  there is a map  $f : M \rightarrow S^n$  of degree 1, i.e. it sends the fundamental class of  $M$  to the fundamental class of  $S^n$ .

**Q4** Find an orientable two-sheeted covering space of the Klein bottle. Which well-known space do you get?

**Q5** (1) Show that  $(\alpha \cap \varphi) \cap \psi = \alpha \cap (\varphi \cup \psi)$  for all  $\alpha \in C_*(X; R)$ ,  $\varphi, \psi \in C^*(X; R)$ . Deduce that the cap product makes  $H_*(X; R)$  a right  $H^*(X; R)$ -module.

(2) Compute the module structure explicitly for  $X$  being an orientable surface of genus  $g$  and  $R = \mathbb{Z}$ . Do the same for  $X$  the Klein bottle.