

## Forcing - Serie 2 - Musterlösung

### Aufgabe 4

Da  $G_3$  und  $G_4$  downwards closed sind, haben wir

$$\{\emptyset, p_0, p_1, p_2, p_3\} \subseteq G_3 \quad \text{und}$$

$$\{\emptyset, p_0, p_2, p_4\} \subseteq G_4.$$

Allgemein gilt  $\tilde{x}[G] = \{\tilde{y}[G] \mid \exists q \in G (\langle \tilde{x}, q \rangle \in \tilde{x})\}$ .

a)  $\tilde{x}_{00} = \{\langle \emptyset, p_0 \rangle\}$ . Also erhalten wir  $\tilde{x}_{00}[G_3] = \{\emptyset[G_3]\} = \{\emptyset\}$ .

Außerdem ist  $\tilde{x}_{00}[G_4] = \{\emptyset[G_4]\} = \{\emptyset\}$ .

b)  $\tilde{x}_{11} = \{\langle \emptyset, p_1 \rangle\}$ . Da  $p_1 \in G_3$  ist, erhalten wir

$$\tilde{x}_{11}[G_3] = \{\emptyset[G_3]\} = \{\emptyset\}$$

Da  $p_1 \notin G_4$  ist, erhalten wir

$$\tilde{x}_{11}[G_4] = \emptyset.$$

c) Es gilt  $\tilde{x}_{01} = \{\langle \emptyset, p_0 \rangle, \langle \emptyset, p_1 \rangle\}$ . Also erhalten wir

$$\tilde{x}_{01}[G_3] = \{\emptyset[G_3], \emptyset[G_3]\} = \{\emptyset\} \quad \text{und}$$

$$\tilde{x}_{01}[G_4] = \{\emptyset[G_4]\} = \{\emptyset\}$$

Weiter haben wir  $\tilde{x}_{01}^{34} = \{\langle \tilde{x}_{01}, p_3 \rangle, \langle \tilde{x}_{01}, p_4 \rangle\}$ . Also

$$\tilde{x}_{01}^{34}[G_3] = \{\tilde{x}_{01}[G_3]\} = \{\{\emptyset\}\} \quad \text{und}$$

$$\tilde{x}_{01}^{34}[G_4] = \{\tilde{x}_{01}[G_4]\} = \{\{\emptyset\}\}.$$

d) Es gilt  $\tilde{x}_{44} = \{\langle \emptyset, p_4 \rangle\}$ . Also erhalten wir

$$\tilde{x}_{44}[G_3] = \emptyset \quad \text{und}$$

$$\tilde{x}_{44}[G_4] = \{\emptyset[G_4]\} = \{\emptyset\}.$$

Weiter haben wir  $\tilde{x}_{44}^{11} = \{\langle \tilde{x}_{44}, p_1 \rangle\}$ . Also

$$\tilde{x}_{44}^{11}[G_3] = \{\tilde{x}_{44}[G_3]\} = \{\emptyset\} \quad \text{und}$$

$$\tilde{x}_{44}^{11}[G_4] = \emptyset$$

e) Es gilt  $\tilde{x}_{11}^{44} = \{\langle \tilde{x}_{11}, p_4 \rangle\}$ . Also erhalten wir

$$\tilde{x}_{11}^{44}[G_3] = \emptyset \quad \text{und} \quad \tilde{x}_{11}^{44}[G_4] = \{\tilde{x}_{11}[G_4]\} = \{\emptyset\}.$$

f) Es gilt  $X_{44}^{34} = \{ \langle X_{44}, P_3 \rangle, \langle X_{44}, P_4 \rangle \}$ . Also erhalten wir

$$X_{44}^{34} [G_3] = \{ X_{44} [G_3] \} = \{ \emptyset \} \text{ und}$$

$$X_{44}^{34} [G_4] = \{ X_{44} [G_4] \} = \{ \emptyset \}.$$

### Aufgabe 5

Siehe Buch Fact 15.1.