Musterlösung Serie 9

Sacks Forcing and Miller Forcing

28-29 (a) We provide an argument that equally works for Sacks and Miller Forcing, where the conditions are respectively defined as perfect and superperfect trees (see Chapter 23 and 25). Let $\mathcal{F}$ be an uncountable almost disjoint family $\mathcal{F} \subset [\omega]^\omega$. For each element $u \in \mathcal{F}$, let T_u be a tree branching (respectively infinitely branching) at level $n \in \omega$ if and only if $n \in u$. We claim that the unconutable family $\{T_u : u \in \mathcal{F}\}$ is an antichain. Indeed, let u and v be distinct elements of $\mathcal{F}$ and consider $T_u \cap T_v$. By definition of almost disjoint, we can fix a level $k \in \omega$ such that for all $l > k$ we have $l \notin u \vee l \notin v$. This implies that $T_u \cap T_v$ can not be branching at any level $l > k$, which implies that T_u and T_v are incompatible, as desired.