

Course: Riemann Surfaces

This PDF will contain all the necessary information for the course. For any further question please write an email to the organizer.

Lecturer: Alessio Cela, HG J 14.4, alessio.cela@math.ethz.ch

Time and location: Monday 10:00-12:00 HG E 21

Goal: The aim of the course is to learn about compact Riemann Surfaces. The main reference Forster's book 'Lectures on Riemann Surfaces'.

Schedule of the Talks

1 Definition of Riemann Surfaces

Speaker: Liang Haoran

Date: 20 Feb

Topic: Definition and first examples of Riemann Surfaces (pages 1-13 of [2]). Solve Exercise 2.1 of [2].

2 Sheaves and Differential Forms

Speaker: Russian Carlo

Date: 27 Feb

Topic: Sheaves on a topological space (pages 40-43 of [2]). Differential forms: definition, definition of cotangent space, cotangent vectors of type $(1,0)$ and $(0,1)$, the residue, meromorphic differential forms, the exterior product, differentiations of forms, elementary properties, pullback (pages 60-68 of [2]).

3 More on Differential Forms

Speaker: Degonda Vincent Andrin

Date: 6 March

Topic: 1-forms, local existence of primitives, existence of a primitive on a covering for closed forms, Periods, Summand of Automorphy, 2-forms, Residue Theorem (pages 69-81 of [2]).

4 Cohomology groups and Dolbeult's Lemma

Speaker: Vetsch Gian

Date: 13 March

Topic: cochains, cocycles, coboundaries, definition of $H^1(\mathcal{U}, \mathcal{F})$, Leray theorem and examples, proof of the fact that $\dim H^1(X, \mathbb{C}) < \infty$ for a compact Riemann Surface X (Exercise 12.2 on page 103 of [2]), Dolbeult's Lemma and applications (pages 96-108 of [2]).

5 Genus of a Riemann Surface

Speaker: Casetta Antonio

Date: 20 March

Topic: Proof of 'A finiteness Theorem', definition of genus, study of meromorphic functions on a genus g Riemann Surface(pages 109-118 of [2]).

6 Cohomology groups of Sheaves and Dolbeult-DeRham theorems

Speaker: Lambert Lorraine Brigitte

Date: 27 March

Topic: The exact Cohomology sequence, Dolbeult's Theorem and DeRham Theorem (pages 119-126 of [2]).

7 Riemann-Roch Theorem and line bundles on genus 0 and 1 curves

Speaker: Langreiter Alexander

Date: 3 April

Topic: Prove Riemann-Roch Theorem (pages 127-131 of [2]). Prove that: two line bundles on \mathbb{P}^1 are isomorphic iff they have the same degree, describe $\mathcal{O}_{\mathbb{P}^1}(d)$ and its sections concretely (see exercise 16.1 on page 131 of [2]). Study line bundles on elliptic curves (solve exercise 16.2 of [2]).

8 Serre Duality and Riemann-Hurwitz formula

Speaker: Tschopp Chiara Olivia

Date: 17 April

Topic: Prove Serre Duality and Riemann-Hurwitz formula (pages 132-140 of [2]).

9 Coverings of \mathbb{P}^1 and embeddings into Projective Space

Speaker: Manea Cosmin

Date: 24 April

Topic: Coverings of \mathbb{P}^1 , maps and embeddings into Projective Space (pages 141-146 of [2]). Solve exercises 17.4 and 17.6 of [2].

10 Harmonic forms and Functions and differential forms with prescribed principal parts

Speaker: Weber Thomas Raphael

Date: 8 May

Topic: Complex conjugation, the $*$ -operator, Harmonic differential forms, scalar product on $\mathcal{E}^{(1)}(X)$, prove that $p_a(X) = b_1(X)$ for any Riemann Surface (pages 153-158 of [2]).

Time permitting cover also the section 'Functions and differential forms with prescribed principal parts': Mittag-Leffler distributions of meromorphic functions, the Wronskian determinant, differential forms with prescribed principal parts (pages 146-152 of [2]).

11 Abel's Theorem

Speaker: Xie Xiaowen

Date: 15 May

Topic: Functions with prescribed divisors, log-differentials, Chains, cycles and homology, Abel's Theorem, application to Doubly-Periodic functions (pages 159-165 of [2]).

12 The Jacobi Inversion Problem

Speaker: Oestmann Lukas and Reinhard Benjamin Anthony

Date: 22 May

Topic: Lattices, Periods lattices, the Jacobi variety and the Picard group, the Jacobi Inversion Problem (pages 166-172 of [2]).

13 Vector bundles on \mathbb{P}^1

Speaker: Adrian Spiess

Date: 29 May

Topic: Prove Grothendieck Theorem about splitting of vector bundle on \mathbb{P}^1 .

References

- [1] E. Arbarello, M. Cornalba, P. Griffiths and J. Harris, *Geometry of algebraic Curves*, Springer-Verlag.
- [2] O. Forster, *Lectures on Riemann Surfaces*, Springer-Verlag.